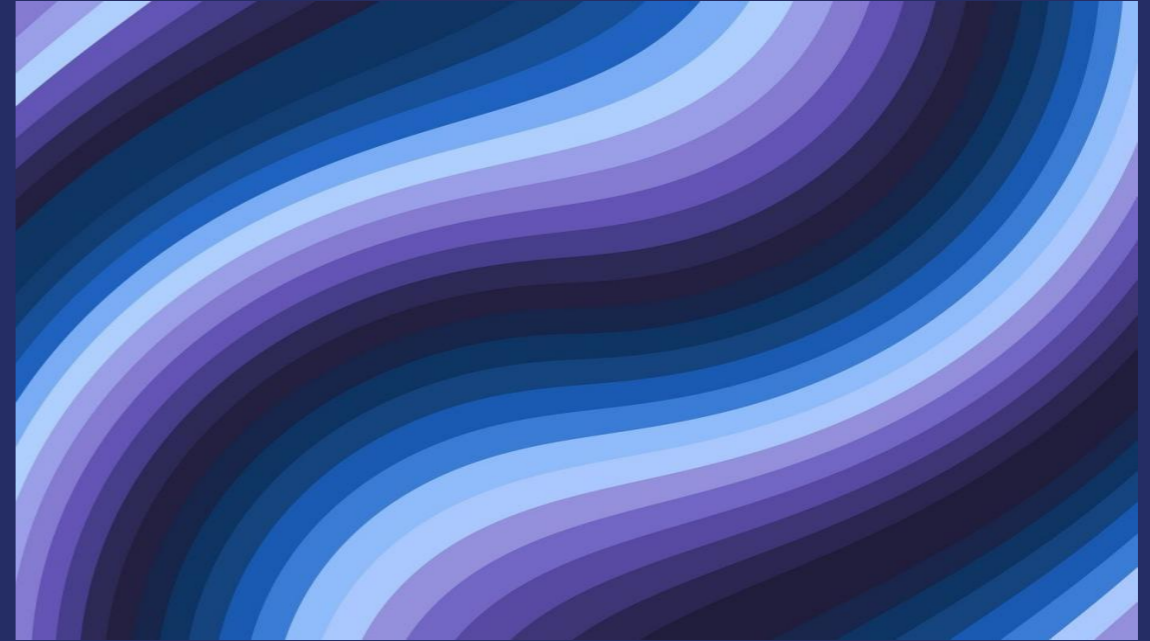


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Linear Control Theory: Computer-
Aided Analytical Methods



LE NGOC GIANG

LINEAR CONTROL THEORY: COMPUTER-AIDED ANALYTICAL METHODS

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**LINEAR CONTROL THEORY:
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Genesis Global Publication

Imprint

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**LINEAR CONTROL THEORY:
COMPUTER-AIDED ANALYTICAL
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by

LE NGOC GIANG

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ABBREVIATION

GM	Gain Margin
LTI	Linear time invariant system
LQR	Linear quadratic regulator
MISO	Multiple input single output
MIMO	Multiple input multiple output
PID	Proportional Integral Derivative
PM	Phase Margin
SS	State-space model
SISO	Singular Input Singular Outputs
SIMO	Singular input multiple output
TF	Transfer function model
ZPK	Zero-pole-gain model

PREFACE

To meet the urgent requirement of standardization and unification of the system of textbooks and documents as a scientific basis for task teaching, learning, and research on effective automatic control theory, we organize and compile the textbook "Linear control theory: Computer-aided analytical methods." The textbook is compiled based on the content of a number of textbooks and documents on automatic control theory, combined with practical experience teaching this subject.

The curriculum includes four chapters:

Chapter 1: Dynamic description of an automatic control system

Chapter 2: Dynamic characteristics of an automatic control system

Chapter 3: Survey of the stability of the automatic control system

Chapter 4: Quality assessment of the automatic control system

Chapter 5: Analyzing control systems using Matlab

The compilation process cannot avoid certain shortcomings. We hope to receive comments from readers and colleagues to make the textbook more and more complete.

Sincerely thank you!

Author

Chapter 1

DYNAMIC DESCRIPTION OF AUTOMATIC CONTROL SYSTEM

The research scope of control theory is broad and encompasses diverse physical systems, including motor control systems, heat furnaces, aircraft, and chemical reactions. Therefore, it is essential to establish a foundation for analyzing and designing control systems with varying physical characteristics. This foundation is grounded in mathematics, where the input-output relationships of linear systems can be expressed through high-order differential equations. However, studying systems based on high-order differential equations presents several challenges, prompting researchers to utilize the method of describing systems using transfer functions for simplicity and effectiveness. In this chapter, we will explore dynamic modeling methods for linear automatic control systems.

1.1. GENERAL OVERVIEW OF AUTOMATIC CONTROL SYSTEMS

1.1.1 Overview of Automatic Control

a) Control

Control is the coordination of a specific process that advances according to a defined rule, ensuring the attainment of a specific objective. Examples include organizing an industrial sewing line, a seafood processing line, an electromechanical tracking system for radar antennas, or an automatic control system to stabilize the flight path of flying devices.

Control involves the following processes: collecting information, processing information, and taking action on the system to align the system's response with the predetermined objective.

Control can be categorized into two types: a process without feedback, known as open-circuit control, and a process with feedback, known as closed-circuit control.

Example 1.1.1: Let's examine the process of controlling the opening and closing of a circuit breaker, as illustrated in Figure 1.1.

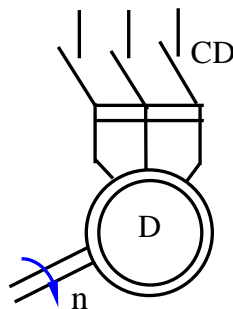


Figure 1.1: The Process of Opening and Closing the Circuit Breaker

When the circuit breaker CD is closed, motor D rotates at speed n , which depends solely on the grid voltage and load. The act of opening and closing the circuit breaker merely alters the operational state of the motor. If the grid voltage remains constant, the action of opening and closing the motor does not change the rotational law governing motor speed n . Despite the fact that the outcomes of closing and opening the circuit breaker can be thoroughly verified through various means, this process represents a control process without feedback, referred to as an open-circuit control process.

In the realm of automation processes, control without feedback lacks scientific or practical significance and is seldom employed. It finds limited use only in situations where high precision is not a requirement.

An essential aspect of automation engineering is the adoption of feedback control systems (closed-circuit control processes). This approach enables the elimination of direct human intervention.

b) Automatic control system

- Automatic control

Automatic control involves the utilization of systems to regulate and manage processes without direct human intervention.

Example 1.1.2: Let's examine a system that regulates the voltage of a DC generator on an aircraft. The schematic diagram of this system is depicted in Figure 1.2.

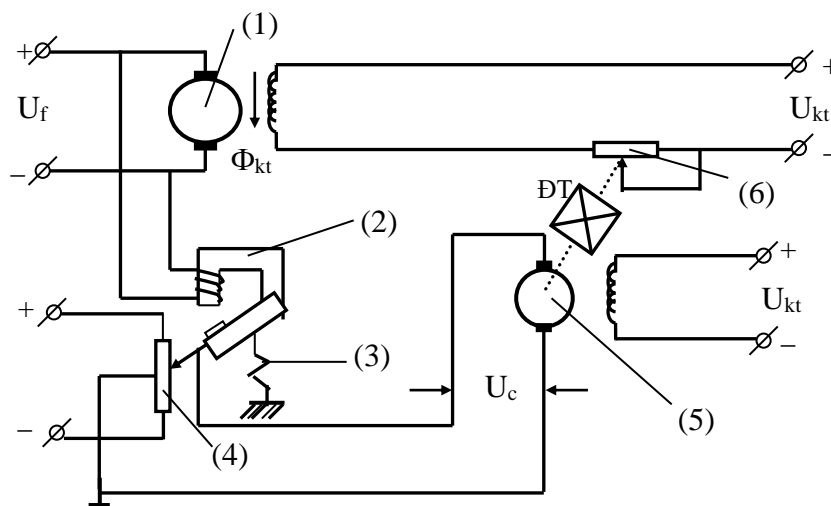


Figure 1.2. Diagram of automatic voltage control of a DC generator on aircraft

In there:

- | | |
|---------------------------|-------------------------------------|
| (1) - DC generator rotor, | (6) - Excitation variable resistor, |
| (2) - Electromagnet, | U_f - Generator voltage, |
| (3) - Spring, | I_{kt} - Excitation current, |
| (4) - Variable resistor, | Φ_{kt} - Excitation flux, |
| (5) - Actuator motor, | U_c - Supply voltage. |

In the normal state, the generator voltage consistently reaches the rated value ($U_f = U_{dm}$). The resistor's brush is in the middle position, resulting in zero voltage supplied to the actuator motor ($U_c = 0$), causing the motor to remain inactive.

Suppose, for some reason, the generator voltage decreases (due to an increased load), leading to a decrease in the electromagnetic attraction of the electromagnet (F_{dt} decreases). Under the influence of the variable resistor's pointer spring, it moves downward, causing the voltage U_c to appear for the executive motor to operate. This action shifts the pointer of the variable resistor (6) to the left (via the gear reducer). Consequently, there is an increase in the field current of the generator, elevating the field flux and, as a result, increasing the generator voltage. This process continues until the generator voltage equals the rated voltage ($U_f = U_{dm}$). Simultaneously, as the generator voltage gradually increases, the electromagnetic attraction force F_{dt} also increases, pulling the variable resistor (4) back to its original position. The voltage U_c supplied to the executive motor gradually decreases to 0. The adjustment process concludes when the generator voltage reaches the rated value.

From the above example, it is evident that automatically controlling the generator voltage involves coordinating the necessary actions to initiate and cease the process, as well as maintaining or altering the voltage levels according to preset requirements, all without direct human involvement.

In summary, automatic control is a synergistic approach that combines the essential coordination needed to maintain or alter process quantities in accordance with predefined requirements, eliminating the need for direct human participation.

- General functional diagram of the automatic control system

An automatic control system is a composite arrangement of technical equipment designed to execute automatic control processes. Examples include missile control systems, unmanned aircraft, and automatic voltage regulation systems.

The general functional diagram of the automatic control system, as illustrated in Figure 1.3.

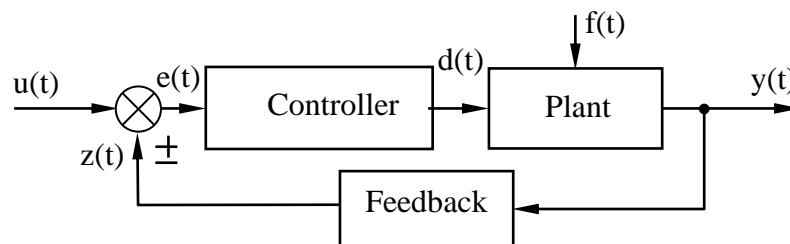


Figure 1.3. General functional diagram of the automatic control system

In which:

Controller - Control device, Plant - Control object ,
Feedback- Feedback device, $e(t)$ - Deviation signal,
 $f(t)$ - Noise signal, $z(t)$ - Feedback signal,
 $y(t)$ - Output signal (output), $d(t)$ - Control signal,
 $u(t)$ - Input signal (input amount).

Control Device (Controller): A control device, also known as a controller, is a combination of technical equipment designed to execute control tasks. Control devices exhibit significant diversity in terms of functions, construction principles, and specific structures.

Control Object: The control object refers to a combination of technical devices wherein specific parameters must be either maintained or altered according to predefined rules. Examples include airplanes, missiles, and the output voltage of voltage stabilizers. Control objects vary widely in terms of effects, impact principles, and structures.

Feedback Device (Sensor): A feedback device is a tool that measures and converts signals from one form of energy to another, which are then transmitted and processed within the system. Examples include devices for measuring angular speed, angular acceleration (such as accelerometers), force measurement (utilizing electrical or pneumatic sensing devices), torque measurement, and temperature sensors.

The fundamental task of control is to generate a signal that acts upon the input of the control object, ensuring that the output quantity attains the desired value. This interaction involves the control device orchestrating actions based on feedback received from the sensor, ultimately achieving the desired control objectives.

Example 1.1.3 . Consider a computer-controlled lathe system as shown in Figure 1.4.

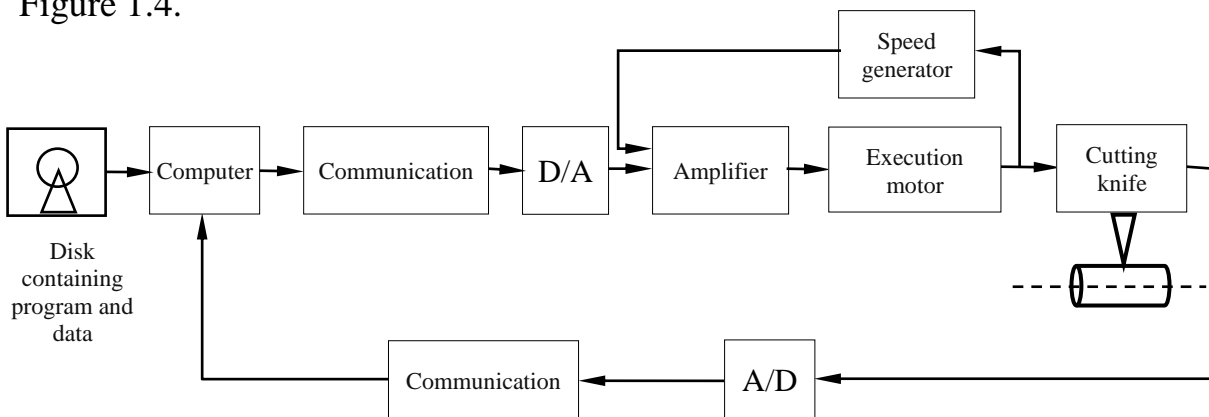


Figure 1.4. Lathe system controlled by computer

The lathe system is under the control of a computer equipped with a control program. This program contains detailed information about the object to be turned, including its shape, size, and other relevant parameters. The computer generates a standard signal, which is then transmitted through the communication port and a digital-to-analog signal converter to reach the amplifier. Here, the analog signal is amplified and directed to the executive motor.

The executive motor, in turn, produces control signals that guide the cutter's operation in accordance with the predetermined requirements. At the system's output, a sensor recognizes the product parameters. This information is then passed through an analog-to-digital signal converter and the communication port, ultimately returning to the computer.

The computer processes the feedback received from the sensor and makes informed decisions for the next course of action. It is important to note that in this diagram, the speed generator functions as a sensor, generating a signal within a secondary feedback loop. This additional feedback loop enhances the control system's capability to respond to changes and optimize performance.

c) Classification of automatic control systems

Automatic control systems are very diverse, they are built on the basis of different principles, different control function tasks,... so there are many methods of classifying automatic control systems. Includes the following classification methods:

- According to the control principle
 - + Deviation control system
 - + Noise compensation control system
 - + Mixed control system.
- According to the law of changing input quantity
 - Predetermined system
 - + Tracking system (automatic system, tracking system)
 - + Programmed control system .
- According to the method of signal transmission and transformation
 - + Continuous system

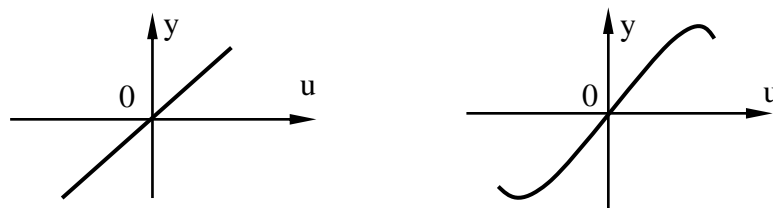


Figure 1.5. Continuous signal graph

+ System interruption

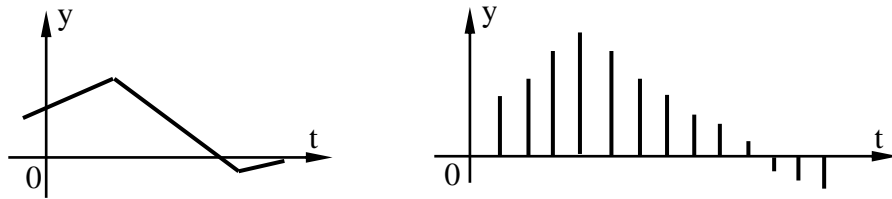


Figure 1.6. Interrupt signal chart

+ Relay system

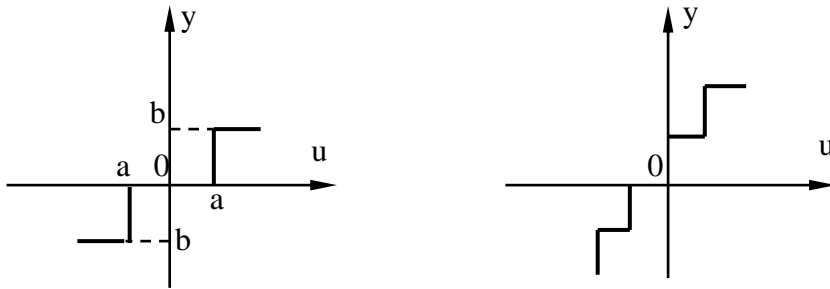


Figure 1.7. Diagram depicting relay system signals

- According to the output quantity
- + One-way system (with one output)
- + Multi-way system (more than one output)
- According to the kinetic equation
- + Linear system
- + Nonlinear system
- According to the error of the system in steady state
- + Systemstatic system (non-zero error)
- + Static system (zero error).

1.1.2. Basic control principles

Control principles can be considered a guideline to design the highest quality and most economically effective control system. Depending on the nature, type of mathematical equation, static characteristics of the control object and most importantly the control task, the principles of automatic control can be very different. The following are the control principles and corresponding system setup methods.

a) Principle of feedback

To achieve high quality control, the system must have two information streams: one from the controller to the control object and one from the control object back to the controller (feedback information stream). Unresponsive

control (open-loop control) cannot achieve high quality, especially in the presence of noise.

Control schemes based on the feedback principle include:

- Noise compensation control

For systems with external impacts on the control object that can be checked and measured, and the properties of the control object and its dynamic characteristics are known, control is carried out on the basis of the results. measurement of external impacts. This type of control is called disturbance compensation control. The control system performs the task of eliminating external impacts (cancelling noise). Its diagram is presented in Figure 1.8.

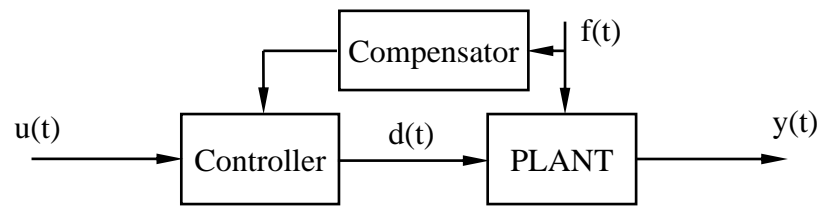


Figure 1.8. Diagram of the noise compensation control system

This is a control principle to achieve the desired output without observing the output signal $y(t)$. For complex systems, noise compensation control cannot provide good quality.

- Deviation leveling control

The controller observes the output signal $y(t)$, compares it with the desired input signal $u(t)$ to calculate the control signal $d(t)$. This control principle adjusts flexibly, eliminates errors, and has good testing and error correction capabilities. This is the basic principle of system control. Its diagram is presented in Figure 1.9.

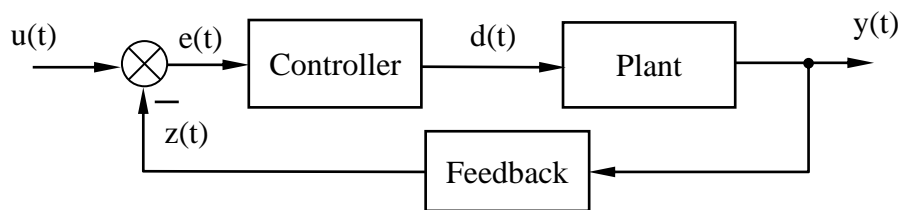


Figure 1.9. Diagram of deviation leveling control system

The content of this principle is to compare the instantaneous state of the control object (result) with the law that needs to be controlled to find the error signal, and use that error signal to control the system.

The quantity to be controlled $y(t)$ along with the control action $u(t)$ is fed into the comparator, where the calculation is performed:

$$e(t) = u(t) - y(t)$$

In which: $e(t)$ - Is the error signal.

If $u(t) = y(t)$ then $e(t) = 0$, the system operates according to a predetermined standard program. When $u(t) \neq y(t)$ then $e(t) \neq 0$, the system operates so that $e(t)$ gradually decreases to zero.

If the control action $u(t)$ does not change sign, but for some reason the quantity to be controlled $y(t)$ changes sign. That is, when negative feedback changes to positive feedback, the system becomes unstable.

The circuit that converts the signal from the output to the input is called the main feedback circuit. In addition to the main feedback circuit, additional component feedback circuits are sometimes added to increase the quality of the system.

- Mix control

Mixed control is a control method that combines both disturbance compensation control and error leveling control. The hybrid control method ensures high precision control. The hybrid control system diagram is depicted in Figure 1.10.

The error leveling and mixed control methods have many advantages, so they are used quite commonly in control techniques.

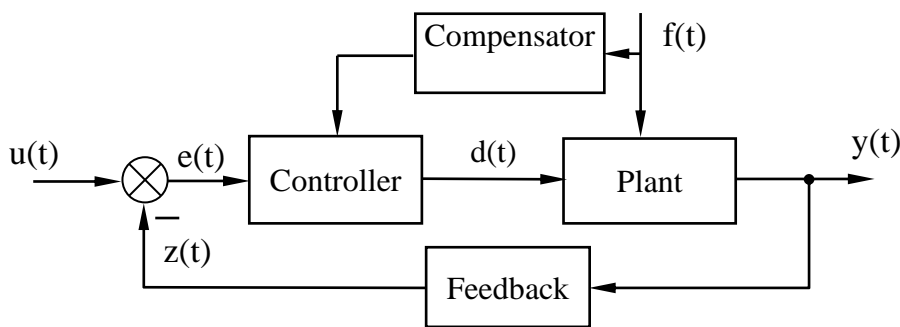


Figure 1.10. Schematic diagram of the hybrid control system

b) The principle of proportional diversity

If you want the control process to have quality, the diversity of the controller must be commensurate with the diversity of the object. The diversity of controllers is reflected in their ability to collect information, store information, transmit information, analyze and process it, make decisions, etc. The meaning of this principle is that it is necessary to design a suitable controller that is compatible with the control object.

c) Principle of External Complementarity

Each control system always exists, operates in a specific environment, and has a close interaction with that environment. Therefore, it is necessary to evaluate external factors affecting the system. The additional principle is that, in addition to admitting that there is an unknown object (the black box) affecting the system, we must control both the system and the black box.

The meaning of the principle of external complementarity is that when designing an automatic system, if you want the system to have high quality, you cannot ignore interference affecting the system.

d) Reserve principle

Because the principle of external supplementation always considers incomplete information, one must be careful of unforeseen events and not use full force under normal conditions. Reserve capital is not used but is needed to ensure the safe operation of the system. Therefore, in some cases, people introduce the reserve principle.

e) Principles of hierarchy

A complex control system needs to build many additional control layers for the center, and then people use the principle of hierarchy to build the control system. The commonly used hierarchical structure is a tree structure, as depicted in Figure 1.11. Most control systems in today's production lines can be divided into three levels:

- Execution level: control the device, read signals from sensors
- Level of coordination
- Organizational and management level.

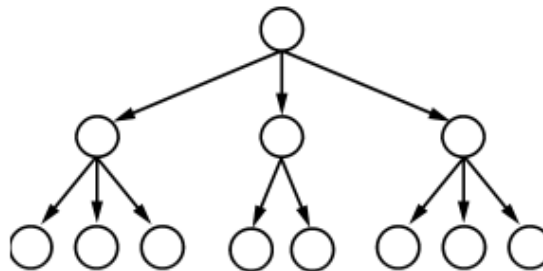


Figure 1.11. Hierarchical control diagram

f) Principle of internal balance

Each system needs to build an internal balance mechanism to be able to self-resolve fluctuations that occur. For example, additional feedback loops can be added to the control system.

1.1.3. Laplace transformation

a) Define Laplace transform

- Forward Laplace transform

Given the function $f(t)$ defined for all $t \geq 0$, the Laplace transform of the function $f(t)$ denoted $F(p)$ is calculated according to the forward transformation formula:

$$F(p) = \int_0^{\infty} f(t) \cdot e^{-pt} dt \quad (1.1)$$

The relationship between function $f(t)$ and $F(p)$ is denoted as:

$$f(t) \xleftrightarrow{L} F(p) \text{ or } F(p) = L \{f(t)\}$$

In which:

t - is the time variable

p - is the Laplace operator variable (complex variable)

$f(t)$ - is the original function

$F(p)$ - is the image function

L - is the symbol for the forward Laplace transform

The Laplace transform exists when the integral in the definition expression (1.1) converges.

- Inverse Laplace transform

If the image function $F(p)$ has an original function $f(t)$, the inverse Laplace transform is calculated by the formula:

$$f(t) = \frac{1}{2\pi j} \cdot \int_{\delta-j\infty}^{\delta+j\infty} F(p) \cdot e^{pt} dp \quad (1.2)$$

The relationship between functions $F(p)$ and $f(t)$ is:

$$f(t) = L^{-1} \{F(p)\}$$

L^{-1} - is the symbol for the inverse Laplace transform.

In practice, using the inverse Laplace transform to find the original function faces many difficulties. We often decompose the image function $F(p)$ into simple fractions and then use the available image origin table to find the original function or use the Heviside formula to find the root function more easily. The Laplace transform is applied to find the output response of the system.

b) Properties of Laplace transform

Assume $f(t)$ and $g(t)$ are two functions with corresponding Laplace transform:

$$F(p) = L\{f(t)\} \text{ and } G(p) = L\{g(t)\}$$

- Linearity

$$L\{Kf(t)\} = K \cdot L\{f(t)\} = KF(p) \quad (1.3)$$

- Stacking calculation

$$L\{f(t) \pm g(t)\} = L\{f(t)\} \pm L\{g(t)\} = F(p) \pm G(p) \quad (1.4)$$

- Image of the derivative

$$L\{f'(t)\} = p.F(p) - f(0)$$

$$L\{f''(t)\} = p^2.F(p) - p.f(0) - f'(0)$$

...

$$L\{f^{(n)}(t)\} = p^n.F(p) - p^{n-1}.f(0) - p^{n-2}.f'(0) - \dots - f^{(n-1)}(0)$$

When the first condition vanishes, that is:

$$f(0) = f'(0) = f''(0) = \dots = f^{(n-1)}(0) = 0 \text{ then}$$

$$L\{f'(t)\} = pF(p)$$

$$L\{f''(t)\} = p^2 F(p)$$

...

$$L\{f^{(n)}(t)\} = p^n.F(p)$$

General formula:

$$L\{f^{(n)}(t)\} = p^n.F(p) \quad (1.5)$$

- Image of integral

If $f(t)$ has an image of $F(p)$ and the initial condition is 0 then

$$L\left\{\int_0^\infty f(t)dt\right\} = \frac{1}{p}F(p) \quad (1.6)$$

- LapLace image of the delay function by a time interval τ

$$L\{f(t - \tau)\} = e^{-\tau p}.L\{f(t)\} = e^{-\tau p}.F(p) \quad (1.7)$$

- LapLace image of convolution

$$L\{f(t) * g(t)\} = L\left\{\int_0^t f(t - T).g(T).dT\right\} = F(p).G(p) \quad (1.8)$$

- Initial value theorem

$$f(0) = \lim_{t \rightarrow 0} f(t) = \lim_{p \rightarrow \infty} p.F(p) \quad (1.9)$$

- Final value theorem

$$f(\infty) = \lim_{t \rightarrow \infty} f(t) = \lim_{p \rightarrow 0} p.F(p) \quad (1.10)$$

Example 1.1.4 . Find Laplace images of the following functions

a) $f(t) = C$

$$F(p) = L\{C\} = \int_0^{\infty} C.e^{-pt}.dt = C.\frac{e^{-pt}}{-p}\Big|_0^{\infty} = \frac{C}{p}$$

b) $f(t) = e^{at}$

$$F(p) = L\{e^{at}\} = \int_0^{\infty} e^{at}.e^{-pt}.dt = \frac{e^{-(p-a)t}}{-(p-a)}\Big|_0^{\infty} = \frac{1}{p-a}$$

c) $f(t) = t$

$$F(p) = L\{t\} = \int_0^{\infty} t.e^{-p.t} dt$$

The integral has the form $\int u.dv = u.v - \int vdu$, where:

$$u = t \rightarrow du = dt ; dv = e^{-pt} dt \Rightarrow v = \int e^{-p.t} dt = -\frac{1}{p}.e^{-p.t}$$

$$\text{So: } F(p) = t.\left(-\frac{1}{p}.e^{-pt}\right)\Big|_0^{\infty} - \int_0^{\infty} \left(-\frac{1}{p}.e^{-pt}\right) dt = -\frac{1}{p^2}.e^{-pt}\Big|_0^{\infty} = \frac{1}{p^2}$$

d) $f(t) = e^{-4t} + \sin(t-2) + t^2.e^{-2t}$

Looking up the image origin table, we have:

$$L\{e^{-4t}\} = \frac{1}{p+4}$$

$$L\{t^2.e^{-2t}\} = \frac{2!}{(p+2)^{2+1}} = \frac{2}{(p+2)^3}$$

$$L\{\sin t\} = \frac{1}{p^2+1}$$

According to the image property of the delay function, then:

$$L\{\sin(t-2)\} = e^{-2p}.L\{\sin t\} = \frac{e^{-2p}}{p^2+1}$$

So according to the superposition property, we get:

$$F(p) = L\{f(t)\} = \frac{1}{p+4} + \frac{e^{-2p}}{p^2+1} + \frac{2}{(p+2)^3}$$

Example 1.1.5 . Find the corresponding root function of the following image function

$$F(p) = \frac{(p+2).e^{-p}}{p^2+4}$$

We have:
$$F(p) = \frac{p.e^{-p}}{p^2+4} + \frac{2.e^{-p}}{p^2+4}$$

From the image source table, then:

$$L^{-1}\left\{\frac{p}{p^2+4}\right\} = \cos 2t \qquad L^{-1}\left\{\frac{2}{p^2+4}\right\} = \sin 2t$$

Applying the delay property we have:

$$L^{-1}\left\{\frac{p.e^{-p}}{p^2+4}\right\} = \cos 2(t-1) \qquad L^{-1}\left\{\frac{2.e^{-p}}{p^2+4}\right\} = \sin 2(t-1)$$

$$f(t) = \cos 2(t-1) + \sin 2(t-1)$$

1.2. DESCRIBE THE SYSTEM USING MATHEMATICAL METHODS

1.2.1. Describe the system using differential equations

a) System working modes

In fact, there are many mathematical methods to describe the dynamic processes of automatic control systems. Because each method has its own advantages and disadvantages, the application scope of each method is also different. Depending on the type of system, use appropriate methods to survey more easily and with higher efficiency.

Automatic control theory is responsible for solving two problems, or in other words, solving two problems: the problem of analysis and the problem of system synthesis. First of all, automatic control theory solves analytical problems and on that basis proposes methods for synthesizing the system. Investigating the properties of the system is based on the mathematical description of the system, that is, establishing the mathematical relationship (equation) between input and output, when the system works in different modes, that is, are: static mode (steady mode) and dynamic mode (transient mode).

- Static mode and static equations

Static mode is the working mode of an automatic system (or element), when the corresponding input and output signals are unchanging functions of time. This means that for a certain fixed value of the input quantity, the system will set a fixed value of the output quantity.

A static equation is a mathematical equation that establishes the relationship between the input and output of the system in static mode.

The static equation has the general form $y = f(u)$. In the case of a linear system, the static equation takes the form:

$$y = Ku \quad (1.11)$$

In which: K is the amplification coefficient (transmission coefficient) of the system.

Figure 1.12 presents the static characteristics of different types of systems (elements), in which the characteristic curve (1) corresponds to the static equation (1.11).

Characteristic (2) corresponds to the case of a negligible nonlinear system; to some extent, we can proceed with linearization and approximate replacement with a linear dependence. We call the system in that case a linearized system and can apply analysis and synthesis methods as for linear systems.

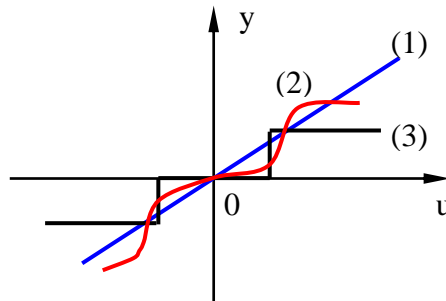


Figure 1.12. Static characteristics of the system (element)

Characteristic (3) is a characteristic of a fully nonlinear automatic system, or a relay-type characteristic. They have their own analysis and synthesis methods and are studied in the nonlinear control system section.

- Dynamic mode and kinematic equations

Dynamic mode is the working mode of an automatic system (element), when the corresponding input and output signals are a function of change over time.

The kinetic equation is a mathematical equation that establishes the relationship between the input and output of the system in dynamic mode.

General form of the kinematic equation:

$$f\{y(t)\} = f\{u(t)\} \quad (1.12)$$

Kinetic equations are usually differential or integral equations. That is, it contains differential or integral components of different orders of input $u(t)$ and output $y(t)$.

By mathematical transformation we can transform an equation containing an integral component into an equation containing only a differential component.

b) Construct the differential equation of the system

- General form of differential equation

Suppose for a system with input $u(t)$ and output $y(t)$ as depicted in Figure 1.13.

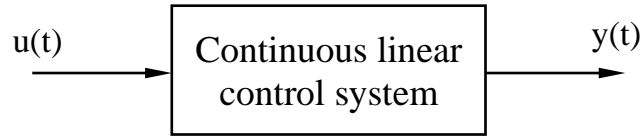


Figure 1.13. Continuous linear automatic control system

Then the differential equation describing the continuous linear control system has the following general form:

$$a_0 \frac{d^n y(t)}{dt^n} + a_1 \frac{d^{n-1} y(t)}{dt^{n-1}} + \dots + a_n y(t) = b_0 \frac{d^m u(t)}{dt^m} + b_1 \frac{d^{m-1} u(t)}{dt^{m-1}} + \dots + b_m u(t) + c_0 \frac{d^l f(t)}{dt^l} + c_1 \frac{d^{l-1} f(t)}{dt^{l-1}} + \dots + c_l f(t) \quad (1.13)$$

In which:

n, m, l are the highest order of output $y(t)$, input $u(t)$, noise $f(t)$; ($n \geq m$).

n is called the degree of the system or the degree of the differential equation.

a_i, b_j, c_k ($i = 0, 1, \dots, n; j = 0, 1, \dots, m; k = 0, 1, \dots, l$) are the coefficients of the equation differential.

Write it down:

$$\sum_{i=0}^n a_i \frac{d^{n-i} y(t)}{dt^{n-i}} = \sum_{j=0}^m b_j \frac{d^{m-j} u(t)}{dt^{m-j}} + \sum_{k=0}^l c_k \frac{d^{l-k} f(t)}{dt^{l-k}} \quad (1.14)$$

When there is only an impact on $u(t)$, then:

$$\sum_{i=0}^n a_i \frac{d^{n-i} y(t)}{dt^{n-i}} = \sum_{j=0}^m b_j \frac{d^{m-j} u(t)}{dt^{m-j}} \quad (1.15)$$

When there is only noise effect $f(t)$, then:

$$\sum_{i=0}^n a_i \frac{d^{n-i} y(t)}{dt^{n-i}} = \sum_{k=0}^l c_k \frac{d^{l-k} f(t)}{dt^{l-k}} \quad (1.16)$$

The research purpose of the automatic control theory course is to find the law of change of output for a certain impact. Therefore, we only need to examine one of two equations (1.15) or (1.16). In fact, we often consider the impact as input quantity $u(t)$, then the differential equation of the automatic control system has the form:

$$a_0 \cdot \frac{d^n y(t)}{dt^n} + a_1 \cdot \frac{d^{n-1} y(t)}{dt^{n-1}} + \dots + a_n \cdot y(t) = b_0 \cdot \frac{d^m u(t)}{dt^m} + b_1 \cdot \frac{d^{m-1} u(t)}{dt^{m-1}} + \dots + b_m \cdot u(t)$$

or

$$\sum_{i=0}^n a_i \frac{d^{n-i} y(t)}{dt^{n-i}} = \sum_{j=0}^m b_j \frac{d^{m-j} u(t)}{dt^{m-j}} \quad (1.17)$$

Equation (1.17) is called the differential equation describing the automatic control system. Solving the differential equation (1.17), we will know the law of change of output $y(t)$ with a certain input $u(t)$.

- Properties of linear systems

If $a_i, b_j = \text{const}$ (or changes insignificantly), then it is a linear system with constant parameters (stationary linear system)

If $a_i, b_j = \text{var}$ then it is a linear system with variable parameters (non-stationary linear system)

If $u(t) = \text{const}; y(t) = \text{const}$ then the system is in static mode. That is, the differential orders are zero, so $a_n y = b_m \cdot u$, therefore $y = ku$; where $k = b_m/a_n$.

This is the static equation (static mode) of the system, that is, the static equation is a special case of the dynamic equation (dynamic mode). If the system is linear, then it has the following properties:

+ Stacking feature

If each output signal $y_i(t)$ of the system is a response to the input signal $u_i(t)$, then the linear combination of input signals that $u(t) = \sum_i u_i(t)$ causes the output signal is a linear combination of the output signals :

$$y(t) = \sum_i y_i(t).$$

The principle of superposition allows for great simplification when studying linear systems, because if the automatic system is subjected to several actions at the same time, we can examine them for each action separately, having That is, research independently of each other. On the other hand, if the system is subjected to a complex impact, we can divide them into a combination of simple impacts (in the form of typical impacts, for example) and study the combination of impacts in and output respectively.

+ Calculate the ratio:

If $y(t)$ is the system's response to the input $u(t)$, then when there is an input $ku(t)$, there will be an output $ky(t)$.

+ Derivative correspondence properties: If $u(t)$ causes the reaction $y(t)$, then $du(t)/dt$ also causes the reaction $dy(t)/dt$.

- Method for building differential equations for the system:

To establish the differential equation of the system, we rely on the physical processes occurring in the system (elements) and the interrelationships between those processes. Each physical process is characterized by certain physical laws. In the process of establishing the kinematic equation, we use known physical equations.

Construction steps are carried out as follows:

Step 1: Based on the physical processes occurring in the element, determine the mathematical formula of that element.

For example, apply Kirchoff's laws and current-voltage relationships on resistors, capacitors, inductors, etc. to electrical elements. Apply Newton's laws—the relationship between friction force and velocity, the relationship between force and spring deformation, to mechanical elements. Apply heat transfer laws, energy conservation laws, etc. to thermal elements.

Step 2: Find the equation of the relationship between the elements through the formula relating physical quantities.

Step 3: Substitute the mathematical formula of each element into the relationship equation and transform it into a canonical differential equation.

The method of describing automatic control systems by differential equations is a classical method, appearing the earliest, but it has an inherent disadvantage: High-order differential equations ($n > 2$) are very difficult to solve, which leads to many difficulties in analyzing and evaluating the stability of the system based on the mathematical model of differential equations. System design based on differential equations is almost impossible in the general case. In addition, the system cannot express its nature in terms of frequency.

Example 1.2.1 Establish a differential equation describing the dynamic properties of the series RLC circuit in Figure 1.14.

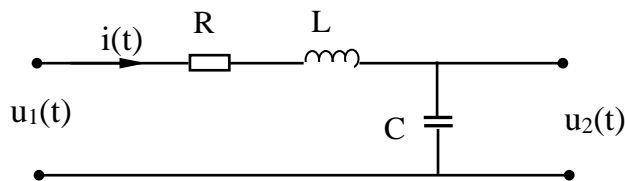


Figure 1.14. Serial RLC circuit

The answer: The voltage applied to the circuit is $u_1(t)$, the output voltage is $u_2(t)$.

Step 1 : Determine the mathematical formula of each element

$$u_2(t) = u_c(t) = \frac{1}{C} \int i(t) dt \Rightarrow i(t) = C \cdot \frac{du_2(t)}{dt}$$

$$u_R(t) = R.i(t) = RC.\frac{du_c(t)}{dt}$$

$$u_L(t) = L.\frac{di(t)}{dt} = LC.\frac{d^2u_c(t)}{dt^2}$$

Step 2 : Find the equation of the relationship between the elements

According to Kirchoff's law: $u_1(t) = u_R(t) + u_L(t) + u_C(t)$

$$u_1(t) = R.i(t) + L.\frac{di(t)}{dt} + u_2(t) \quad (1.18)$$

Step 3 : Substitute the formula into the equation and transform it to a canonical differential equation

Substituting the mathematical formula of the elements into (1.18) and transforming, we get:

$$LC\frac{d^2u_2(t)}{dt^2} + RC\frac{du_2(t)}{dt} + u_2(t) = u_1(t) \quad (1.19)$$

Equation (1.19) is a differential equation that describes the dynamic properties of a given circuit.

1.2.2. Describe the system using a transfer function

a) Define transfer function

Investigating systems based on differential equations (1.17) is very difficult, especially for equations with degrees greater than two. Therefore, we need another mathematical representation to make studying automatic control systems easier. Through the Laplace transform, we can do this.

The basic content of applying the Laplace operator to investigate the dynamic properties of the system is to convert from the d/dt differential operator plane to the Laplace operator plane.

If the system has an input $u(t)$ and an output $y(t)$, then the differential equation describing the system has the form:

$$a_0\frac{d^n y(t)}{dt^n} + a_1\frac{d^{n-1}y(t)}{dt^{n-1}} + \dots + a_n y(t) = b_0\frac{d^m u(t)}{dt^m} + b_1\frac{d^{m-1}u(t)}{dt^{m-1}} + \dots + b_m u(t) \quad (1.20)$$

Assuming the initial condition is 0, applying the image property of derivatives to transform Laplace on both sides of equation (1.20), we get:

$$(a_0 p^n + a_1 p^{n-1} + \dots + a_n).Y(p) = (b_0 p^m + b_1 p^{m-1} + \dots + b_m).U(p) \quad (1.21)$$

Establishing the ratio between the output image function and the input image function we have:

$$W(p) = \frac{Y(p)}{U(p)} = \frac{b_0 p^m + b_1 p^{m-1} + \dots + b_m}{a_0 p^n + a_1 p^{n-1} + \dots + a_n} \quad (1.22)$$

$W(p)$ is called the transfer function of the element or system.

Definition : The transfer function of an element or system is the ratio between the Laplace image function of the output quantity and the Laplace image function of the input quantity when the initial condition is zero.

It should be emphasized that although the transfer function is defined as the ratio between the Laplace image function of the output and input quantities, the transfer function does not depend on the output and input quantities, but only on the order and parameters of the system. system (note the right-hand side of expression (1.22)), so we can use a transfer function to describe the system. In other words, based on the transfer function, we can evaluate the characteristics of the automatic control system. Describing an automatic control system using differential equations (1.17) or transfer functions(1.22) is completely equivalent, but investigating the system based on the transfer function is much easier because the transfer function is an algebraic fraction that has neither integral nor differential calculus, in which:

Polynomial in the numerator : $B(p) = b_0 p^m + b_1 p^{m-1} + \dots + b_{m-1} p + b_m$.

The solution of the equation $B(p) = 0$ is called Zero (zero point).

The polynomial in the denominator: $A(p) = a_0 p^n + a_1 p^{n-1} + \dots + a_n$ is called a characteristic polynomial.

$A(p) = 0$ is called the characteristic equation .The solution of the characteristic equation is called the Pole. Through the characteristic equation, the stability of the system can be considered.

Example 1.2.2 .Establish a transfer function that describes the dynamic properties of the series RLC circuit in Figure 1.14.

The answer:

In example 1.2.1, the differential equation of the RLC circuit has been established. To find the transfer function, we transform the Laplace transform on both sides of equation (1.19) with the initial condition equal to 0:

$$\left(LCp^2 + RCp + 1 \right) \cdot U_2(p) = U_1(p) \quad (1.23)$$

Setting the ratio between the output signal and the input signal we get:

$$W(p) = \frac{U_2(p)}{U_1(p)} = \frac{1}{LCp^2 + RCp + 1} \quad (1.24)$$

Expression (1.24) is the transfer function of a series RLC circuit.

b) Meaning and method of constructing the transfer function

- Meaning of transfer function

The basic content of the course Automatic Control Theory is an analysis of system dynamics based on the mathematical model of the transfer function. By observing the transfer function, we will recognize the system structure. Through the transfer function, we can also determine the initial value and final value (according to the properties of the Laplace transform), determine the stability of the system, and determine the static gain coefficient of the system.

$$k_t = \lim_{p \rightarrow 0} W(p) \quad (1.25)$$

From the expression of the transfer function (1.17), we can determine the output response of the system over time:

$$Y(p) = W(p) \cdot U(p);$$

To find the output response $y(t)$, use the inverse Laplace transform:

$$y(t) = L^{-1}[Y(p)] \quad (1.26)$$

Example 1.2.3 . A system is described by the following differential equation

$$\frac{d^2 y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = 3 \cdot \frac{du(t)}{dt} + u(t)$$

Find the law of change of output $y(t)$? Knowing $u(t) = 1$.

The answer: Perform the Laplace transform with zero initial condition.

$$Y(p) = \frac{3p+1}{p^2-p-2} \cdot \frac{1}{p}$$

Applying the Heviside formula, we get the output as:

$$y(t) = -\frac{1}{2} - \frac{2}{3} \cdot e^{-t} + \frac{7}{6} \cdot e^{2t}$$

- Method of building a transfer function

Step 1: Set up a differential equation that describes the relationship between the input quantity and the output quantity of the system by applying laws, theorems, relationships,... between the elements in the system.

Step 2: Laplace transform both sides of the differential equation just established in step 1 with the initial condition equal to zero, then set the ratio of the output image function to the input image function, we get the transfer function we need to find.

Example 1.2.4. Give the differential equation of the missile angular coordinate tracking system as follows

$$T_2 \frac{d^2 y(t)}{dt^2} + \frac{dy(t)}{dt} = KT_1 \frac{du(t)}{dt} + Ku(t)$$

Find the transfer function of the above missile angular coordinate system?

The answer: Using the properties of the Laplace transform, transforming the image of the above differential equation we get:

$$T_2 p^2 Y(p) + Y(p) = KT_1 p U(p) + KU(p)$$

Establishing the ratio between the output quantity and the input quantity, we can find the transfer function of the missile angular coordinate tracking system by:

$$W(p) = K \frac{(1 + pT_1)}{p(1 + pT_2)}$$

From the above examples, we can see that describing the system using a transfer function has brought great benefits in analyzing and investigating the dynamic properties of the system. To avoid directly solving the differential equation, people convert the differential operator to the Laplace operator to solve the algebraic equation more easily, then use the intermediate step to find the rule of the output, which is convenient and simple. than.

However, to calculate the transfer function, the initial condition must be zero, assuming the conditions are zero to simplify the use of the transfer function to study the dynamic nature of the system . The concept of transfer function is only used for continuous linear systems and cannot be used for nonlinear systems. In addition, studying the system in the frequency domain is quite complicated.

1.3. DESCRIBE THE SYSTEM USING A STRUCTURE DIAGRAM

1.3.1. General overview of the structural diagram

a) Structure diagram

- Concept

Structural diagram is a diagram showing the connection between dynamic stages of an automatic control system.

The structural diagram of the system can be obtained from the functional diagram by describing the dynamic properties of the functional elements by their transfer functions.

- How to represent structural diagrams

+ Diagrammatic representation

The dynamic links are linked together to form the main effect circuit and feedback circuit (main feedback, partial feedback).

Each kinematic step is denoted by a rectangle, in which is written the transfer function of that step. Signals are written as operators and have arrows indicating the direction of the signal.

Example 1.3.1 . An automatic control system has a structure diagram as shown in Figure 1.15.

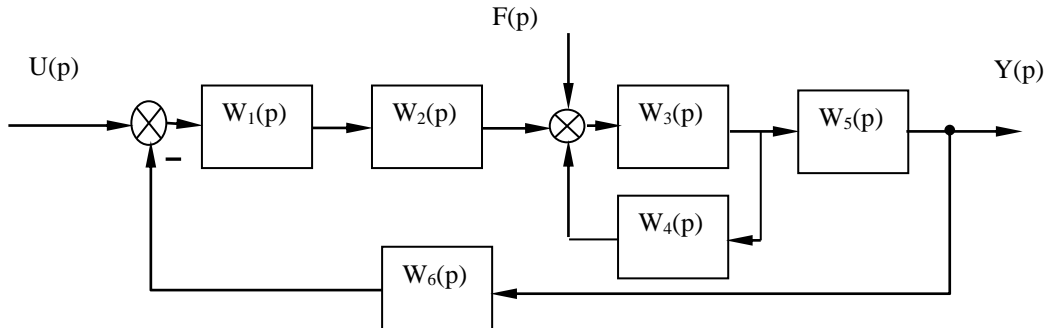
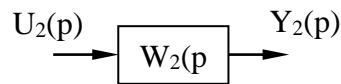


Figure 1.15. Structure diagram of an automatic control system

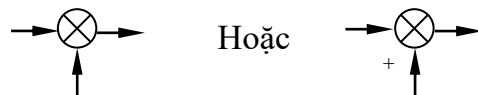
+ Some conventions in structural diagrams

Function block: Each function block has one input signal and one output signal with a transfer function equal to:



$$W_2(p) = \frac{Y_2(p)}{U_2(p)}$$

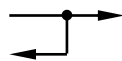
Additive compare button: Both input signals to the button have an additive effect.



Minus compare button (subtractor): One input signal to the button has a plus effect, one input signal has a minus effect.



Branch button:



b) Basic connections in the structural diagram

The structure diagram of an automatic control system can include the following three basic connections: serial, parallel and feedback.

- Serial connection

Serial connection is a way to connect the diagram when the output of the previous stage is the input of the next stage.

Suppose a system has n links connected in series, as depicted in Figure 1.16.

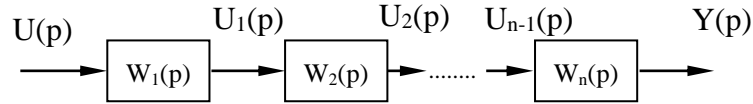


Figure 1.16. The system has n links connected in series

$$Y(p) = U_{n-1}(p) \cdot W_n(p) = U_{n-2}(p) \cdot W_{n-1}(p) \cdot W_n(p)$$

$$Y(p) = U(p) \cdot W_1 \cdot W_2 \dots W_n(p)$$

$$W_{nt}(p) = \frac{Y(p)}{U(p)} = \prod_{i=1}^n W_i(p) \quad (1.27)$$

Thus, the transfer function of a system with n series connections is equal to the product of the component transfer functions.

- Parallel connection

Parallel connection is a way to connect the diagram when the outputs of the stages have the same signal direction and are sent to the same comparator.

Suppose a system has n parallel connections, as depicted in Figure 1.17.

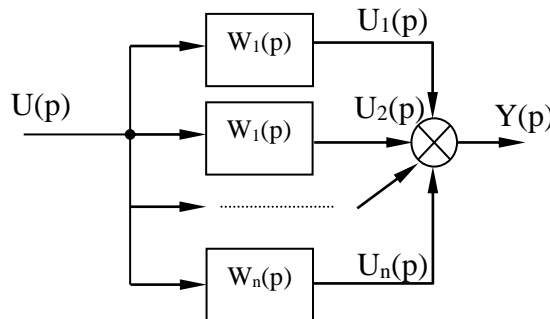


Figure 1.17. The system has n parallel connections

$$Y(p) = U_1(p) + U_2(p) + \dots + U_n(p)$$

$$Y(p) = W_1(p) \cdot U(p) + W_2(p) \cdot U(p) + \dots + W_n(p) \cdot U(p)$$

$$Y(p) = \{W_1(p) + W_2(p) + \dots + W_n(p)\} \cdot U(p)$$

$$W_{ss}(p) = \sum_{i=1}^n W_i(p) \quad (1.28)$$

Thus, the transfer function of a system with n parallel connections is equal to the sum of the component transfer functions.

- Feedback connection

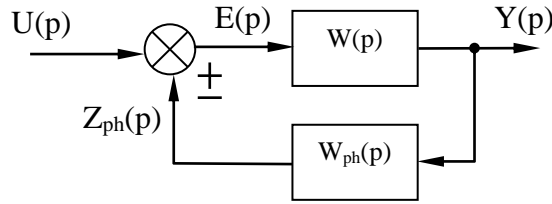


Figure 1.18. The system has a feedback link

$$Y(p) = E(p) \cdot W(p) = [U(p) \pm Z_{ph}(p)] \cdot W(p) = [U(p) \pm Y(p) \cdot W_{ph}(p)] \cdot W(p)$$

$$[1 \mp W(p) \cdot W_{ph}(p)] \cdot Y(p) = W(p) \cdot U(p)$$

$$W_k(p) = \frac{W(p)}{1 \mp W(p) \cdot W_{ph}(p)} \quad (1.29)$$

In expression (1.29) the minus sign corresponds to positive feedback, the plus sign corresponds to negative feedback of the diagram depicted in figure 1.18.

If the unit response $W_{ph}(p) = 1$, then:

$$W_k(p) = \frac{W(p)}{1 \mp W(p)} \quad (1.30)$$

1.3.2. Rules for transforming structural diagrams

a) Overview of the structural diagram with cross-links

- Concept: A cross-linked structural diagram is a diagram on which signals from one closed-loop circuit are transferred to another closed-loop circuit.

Example 1.3.2. Consider a structural diagram with three closed-loop circuits, as shown in Figure 1.19.

The signal taken from node A of closed loop I is transferred to closed loop III creating a cross-link. To find the equivalent transfer function of the system, it is necessary to remove this cross-link.

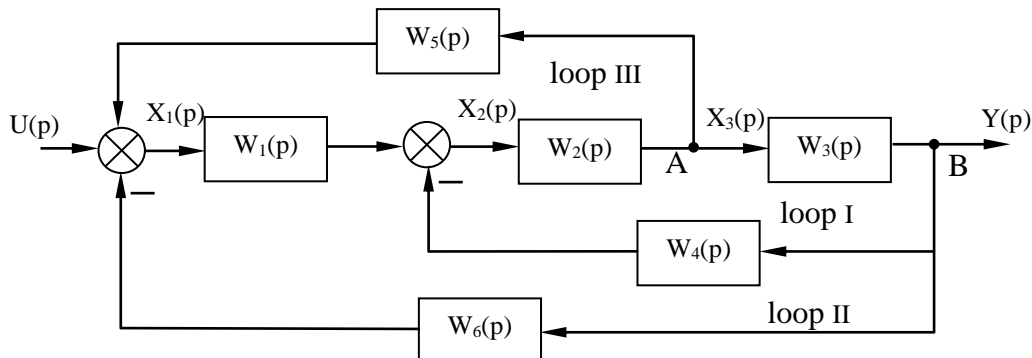


Figure 1.19. The system has cross-linkages

- General rules for transforming structural diagrams

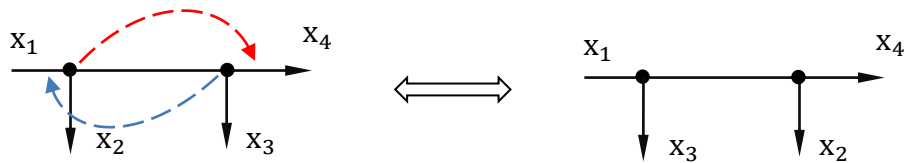
Purpose: To find the equivalent transfer function of the whole system.

Requirements: The input and output signals of the transformed diagram must not change their values, so that the original diagram and the transformed diagram are equivalent. In other words, the transfer function of the original diagram and the transformed diagram must be equal.

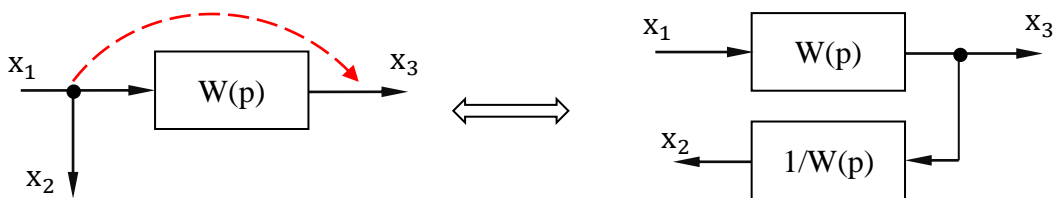
Task: Must switch buttons, comparators or stages so that the signals between the loops are independent.

b) Basic transformation rules

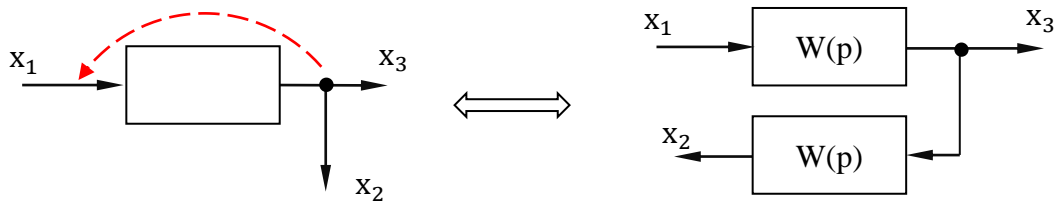
Rule 1: Relocate two branch nodes



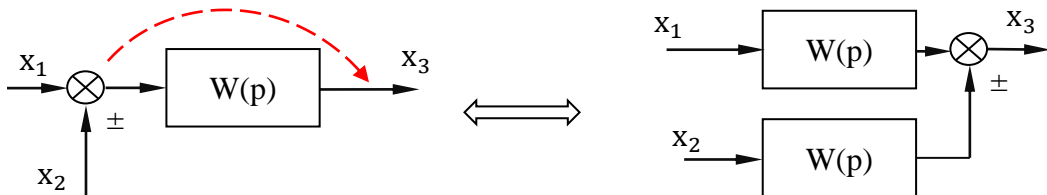
Rule 2: Move the branch node from front to back one block



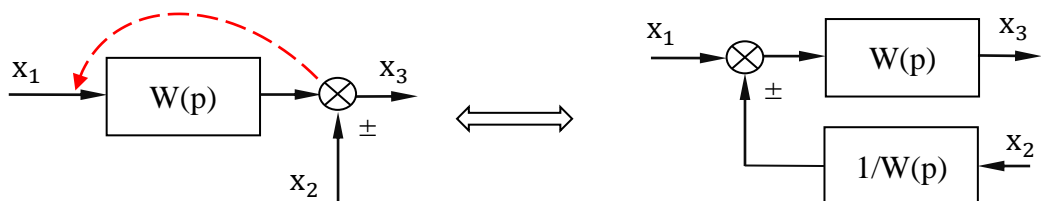
Rule 3: Move the branch node from back to front one block



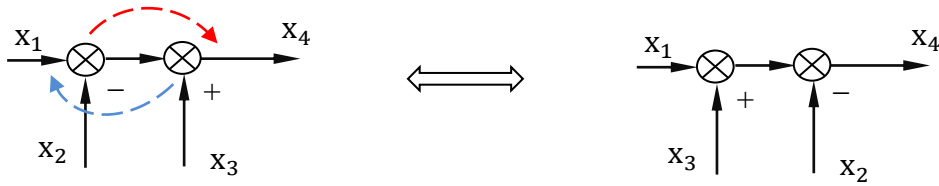
Rule 4: Move the comparator from front to back one block



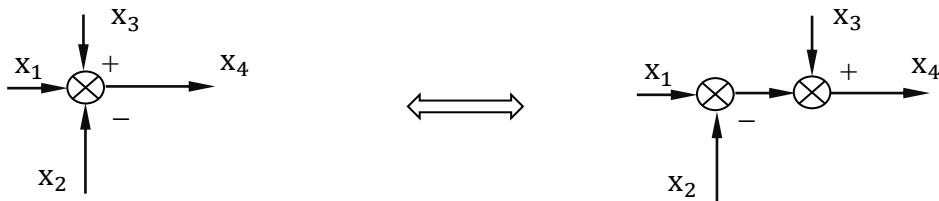
Rule 5: Move the comparator from back to front one block



Rule 6: Switch two comparators



Rule 7: Split a comparator into two comparators



Note, do not switch a branch node through a comparator, and do not switch two comparators when there is a branch node in the middle.

1.3.3. Types of transfer functions of structural diagrams

Suppose an automatic control system is described by the structure diagram shown in Figure 1.20.

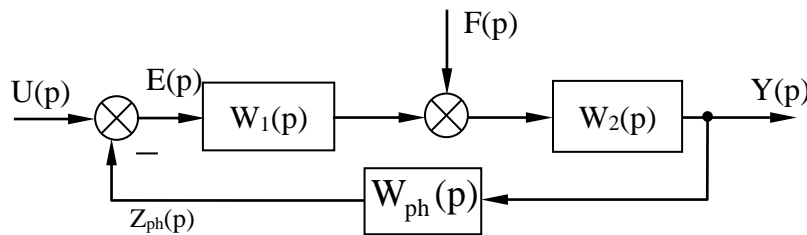


Figure 1.20. The system has two effects

In which: $U(p)$ is the input signal, $Y(p)$ is the output signal, $F(p)$ is the noise signal, $E(p)$ is the error signal.

From the structure diagram in Figure 1.20, we can identify the following four basic types of transfer functions.

a) *Open system transfer function*

When disconnecting the feedback connection at point A and considering zero noise $F(p) = 0$, then the remaining diagram is as shown in Figure 1.21.

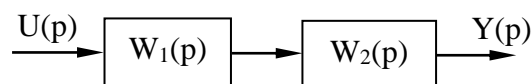


Figure 1.21. Open system diagram

$$W_h(p) = \frac{Y(p)}{U(p)} = \frac{Y(p)}{E(p)} = W_1(p) \cdot W_2(p) \quad (1.31)$$

b) Closed system transfer function

Assuming zero interference $F(p) = 0$, then the diagram will be as shown in Figure 1.22.

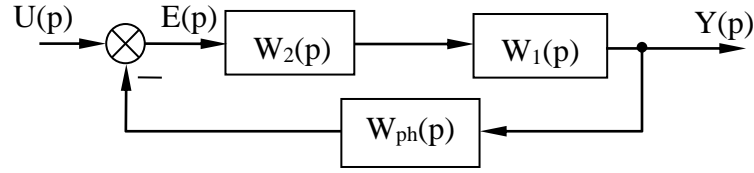


Figure 1.22. Closed system diagram

$$W_k(p) = \frac{Y(p)}{U(p)} = \frac{W_h(p)}{1 + W_{ph}(p) \cdot W_h(p)} \quad (1.32)$$

When the unit response $W_{ph}(p) = 1$, then:

$$W_k(p) = \frac{Y(p)}{U(p)} = \frac{W_h(p)}{1 + W_h(p)} \quad (1.33)$$

c) Transfer function by error

The error transfer function is the ratio of the error image function $E(p)$ to the input image function $U(p)$ with the initial condition being zero and the noise effect being zero.

The error transfer function is denoted by $W_E(p)$:

$$W_E(p) = \frac{E(p)}{U(p)}$$

Using the structure diagram in Figure 1.22, we have

$$E(p) = U(p) - Z_{ph}(p) = U(p) - W_{ph}(p) \cdot Y(p)$$

Should:

$$W_E(p) = \frac{U(p) - W_{ph}(p) \cdot Y(p)}{U(p)} = 1 - W_{ph}(p) \cdot W_k(p)$$

$$W_E(p) = \frac{1}{1 + W_{ph}(p) \cdot W_h(p)} \quad (1.34)$$

If the unit response $W_{ph}(p) = 1$, then:

$$W_E(p) = 1 - \frac{W_h(p)}{1 + W_h(p)} = \frac{1}{1 + W_h(p)} \quad (1.35)$$

d) Transfer function under noise

The noise transfer function is the ratio of the output image function $Y(p)$ to the noise image function $F(p)$ with the initial condition being zero and the input being zero.

The transfer function under noise is denoted by $W_F(p)$:

$$W_F(p) = \frac{Y(p)}{U(p)}$$

At this time, $U(p) = 0$, the structural diagram is described as shown in Figure 1.23.

Using expression (1.29) for the feedback link we get:

$$W_F(p) = \frac{Y(p)}{F(p)} = \frac{W_2(p)}{1 + W_1(p) \cdot W_2(p) \cdot W_{ph}(p)} \quad (1.36)$$

$$W_F(p) = \frac{W_2(p)}{1 + W_h(p) \cdot W_{ph}(p)} \quad (1.37)$$

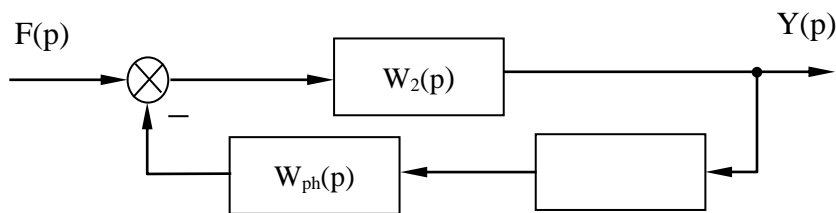


Figure 1.23. System diagram according to interference effects

Example 1.3.3. Simplify the structural diagram, find the transfer functions $W_h(p)$, $W_k(p)$, $W_E(p)$ of the system described in Figure 1.24.

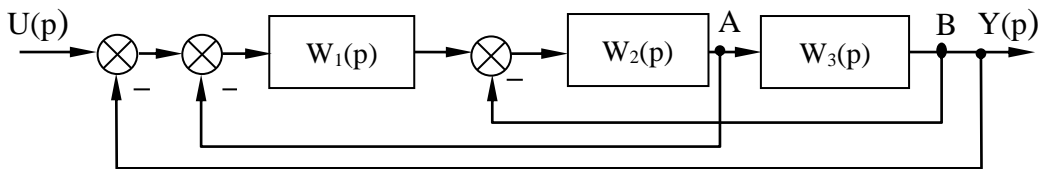


Figure 1.24. The system has cross-linkages

The answer: Move the branch node from before $W_3(p)$ to after $W_3(p)$, then move the position between two nodes A and B as shown in Figure 1.25.

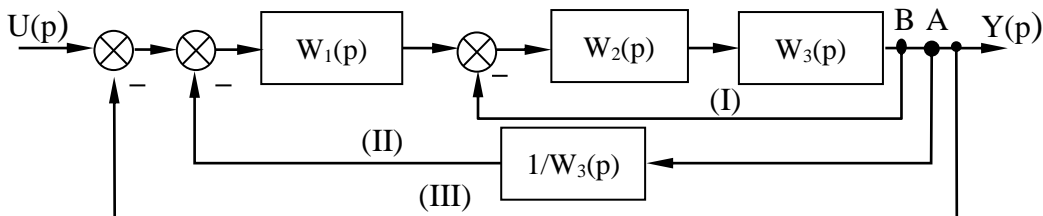


Figure 1.25. The system after transformation

Considering closed loop (I), its transfer function is

$$W_I(p) = \frac{W_2(p) \cdot W_3(p)}{1 + W_2(p) \cdot W_3(p)}$$

Considering closed loop (II), its transfer function is

$$W_{II}(p) = \frac{W_1(p)W_2(p)}{1 + W_1(p)W_2(p) \frac{1}{W_3(p)}}$$

Substituting the value of $W_1(p)$ into the above expression, we find the open system transfer function by:

$$W_h(p) = W_{II}(p) = \frac{W_1(p)W_2(p)W_3(p)}{1 + W_1(p)W_2(p) + W_2(p)W_3(p)}$$

The transfer function conveys a closed system by:

$$W_k(p) = W_{III}(p) = \frac{W_h(p)}{1 + W_h(p)}$$

$$W_k(p) = \frac{W_1(p)W_2(p)W_3(p)}{1 + W_1(p)W_2(p) + W_2(p)W_3(p) + W_1(p)W_2(p)W_3(p)}$$

The transfer function in terms of error is equal to:

$$W_E(p) = \frac{1}{1 + W_h(p)} = \frac{1}{1 + W_1(p)W_2(p) + W_2(p)W_3(p) + W_1(p)W_2(p)W_3(p)}$$

1.4. CHAPTER 1 EXERCISES

1.4.1. Sample solution exercises

Exercise 1: Use the Laplace transform to solve the following differential equation

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 5u(t)$$

With $u(t)$ being the unit step function, and the initial condition: $y(0) = -1$; $y'(0) = 2$.

The answer: Transforming the Laplace transform on both sides of the original differential equation, we have:

$$p^2Y(p) - py(0) - y^{(1)}(0) + 3pY(p) - 3y(0) + 2Y(p) = \frac{5}{p}$$

$$\Rightarrow Y(p) = \frac{-p^2 - p + 5}{p(p^2 + 3p + 2)} = \frac{-p^2 - p + 5}{p(p+1)(p+2)}$$

Decompose the function $Y(p)$ into a sum of simple fractions:

$$Y(p) = \frac{-p^2 - p + 5}{p(p+1)(p+2)} = \frac{A}{p} + \frac{B}{p+1} + \frac{C}{p+2}$$

Using the coefficient balance method, we can determine the coefficients A, B, C:

$$Y(p) = \frac{-p^2 - p + 5}{p(p+1)(p+2)} = \frac{5}{2p} - \frac{5}{p+1} + \frac{3}{p+2}$$

Looking up the Laplace transform table, we get:

$$y(t) = \frac{5}{2} - 5e^{-t} + \frac{3}{2}e^{-2t} \quad ; t \geq 0$$

Exercise 2: Find the transfer function of the operational amplifier circuit described in Figure 1.26.

The answer: According to the circuit diagram, the current relationship is:

$$i_1(t) = i_2(t) + i_3(t)$$

In there:

$$i_1(t) = \frac{u_v(t) - u'(t)}{R_1};$$

$$i_2(t) = C \frac{d(u'(t) - u_r(t))}{dt};$$

$$i_3(t) = \frac{u'(t) - u_r(t)}{R_2};$$

Inferred

$$\frac{u_v(t) - u'(t)}{R_1} = C \frac{d(u'(t) - u_r(t))}{dt} + \frac{u'(t) - u_r(t)}{R_2}$$

The characteristic of an operational amplifier is that the input impedance is very large (can be considered as infinity), so the current entering the operational amplifier $i' = 0$ or $u' = 0$, therefore:

$$\frac{u_v(t)}{R_1} = -C \frac{du_r(t)}{dt} - \frac{u_r(t)}{R_2}$$

Two-sided Laplace transform:

$$\frac{U_v(p)}{R_1} = -\frac{R_2 C p + 1}{R_2} U_r(p)$$

Setting the ratio between the output signal and the input signal, we get the transfer function:

$$W(p) = \frac{U_r(p)}{U_v(p)} = -\frac{R_2}{R_1} \cdot \frac{1}{R_2 C p + 1}$$

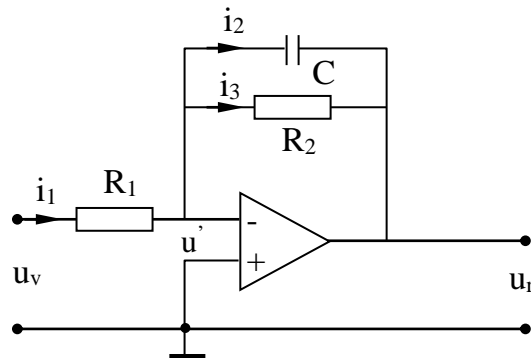


Figure 1.26. Operational amplifier circuit

1.4.2. Self-explanatory exercises

1. Find the Laplace image function of the following original function

$$b) f(t) = \begin{cases} 0 & \text{khi } t < 0 \\ e^{-0,4t} \cos 12t & \text{khi } t \geq 0 \end{cases}$$

$$c) f(t) = \begin{cases} 0 & \text{khi } t < 0 \\ 3 \sin(5t + 45^\circ) & \text{khi } t \geq 0 \end{cases}$$

$$d) f(t) = \begin{cases} 0 & \text{khi } t < 0 \\ 0,33(1 - \cos 2t) & \text{khi } t \geq 0 \end{cases}$$

2. Find the origin function of the following Laplace image functions

$$a) F(p) = \frac{p+3}{(p+1)(p+2)}$$

$$b) F(p) = \frac{2p+12}{p^2+2p+5}$$

$$c) F(p) = \frac{p^2+2p+3}{(p+1)^3}$$

Answer:

$$a) f(t) = 2e^{-t} - e^{-2t} \quad \text{khi } t \geq 0$$

$$b) f(t) = 5e^{-t} \sin 2t + 2e^{-t} \cos 2t \quad \text{khi } t \geq 0$$

$$c) f(t) = (1+t^2)e^{-t} \quad \text{khi } t \geq 0$$

3. Find the origin function of the following Laplace image functions:

$$a) F(p) = \frac{p+2}{p(p+1)^2(p+3)}$$

$$b) F(p) = \frac{5(p+2)}{p^2(p+1)(p+3)}$$

$$c) F(p) = \frac{(p+2) \cdot e^{-p}}{p^2+4}$$

Answer:

$$a) f(t) = \frac{2}{3} - \frac{3}{4}e^{-t} - \frac{1}{2}te^{-t} + \frac{1}{12}e^{-3t}$$

$$b) f(t) = -\frac{25}{9} + \frac{10}{3}t + \frac{5}{2}e^{-t} + \frac{5}{18}e^{-3t}$$

$$c) f(t) = \cos 2(t-1) + \sin 2(t-1)$$

4. Use the Laplace transform to solve the following differential equation

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 5y(t) = 0$$

With the initial condition $y(0) = 1$, $y'(0) = \left. \frac{dy(t)}{dt} \right|_{t=0} = 0$.

$$\text{Answer: } y(t) = e^{-t} \cos 2t + \frac{1}{2}e^{-t} \sin 2t$$

5. Find the transfer function of the operational amplifier scheme shown in Figure 1.27.

Figure 1.27.

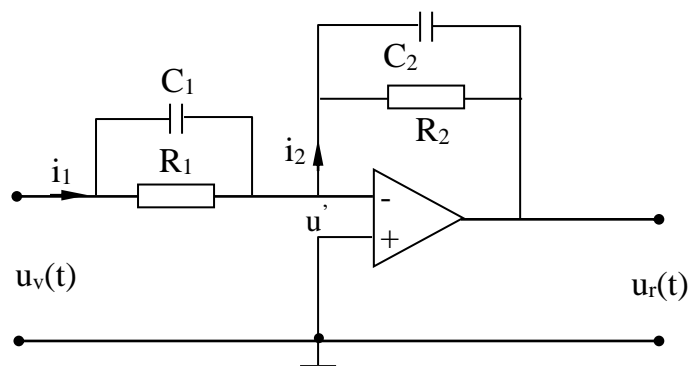


Figure 1.27. Circuit diagram for operational amplifier

$$\text{Answer: } W(p) = \frac{U_r(p)}{U_v(p)} = -\frac{Z_2}{Z_1} = -\frac{R_2}{R_1} \frac{R_1 C_1 p + 1}{R_2 C_2 p + 1} = -\frac{C_1}{C_2} \frac{p + \frac{1}{R_1 C_1}}{p + \frac{1}{R_2 C_2}}$$

6. Find the transfer function of the circuit diagram in Figure 1.28.

$$\text{Answer: } W(p) = \frac{Y(p)}{U(p)} = \frac{(R_1 C_1 p + 1)(R_2 C_2 p + 1)}{(R_1 C_1 p + 1)(R_2 C_2 p + 1) + R_2 C_1 p}$$

7. For a navigation system between a satellite (flying object) and the sun, with a closed system negative feedback the unit has an open system transfer function as follows:

$$W(p) = \frac{K(p + 20)}{p(p^2 + 24p + 144)}$$

Find the differential equation of the closed system.

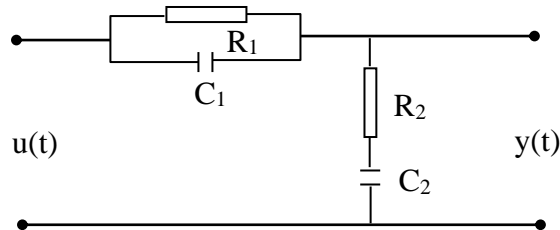


Figure 1.28. Electrical circuit diagram

8. Determine the transfer function of the system with the structural diagram shown in Figure 1.29.

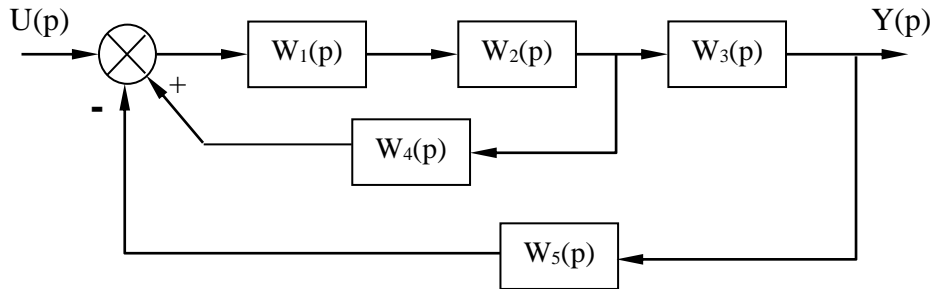


Figure 1.29. Structural diagram of a system

Answer :
$$W(p) = \frac{U(p)}{Y(p)} = \frac{W_1 W_2 W_3}{1 - W_1 W_2 W_4 + W_1 W_2 W_3 W_5}$$

9. Determine the transfer function of the system with the structural diagram shown in Figure 1.30.

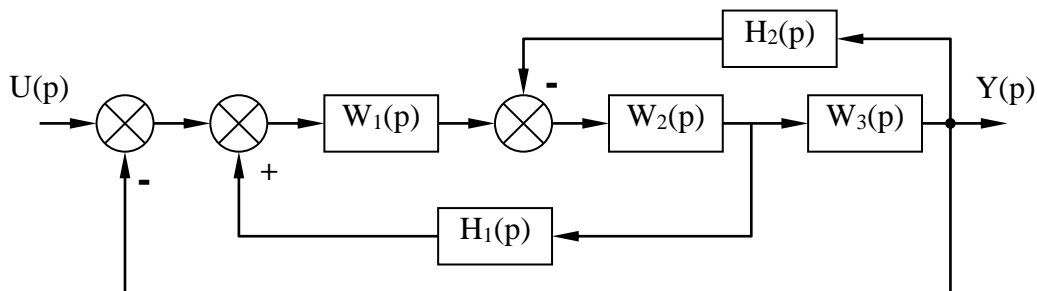


Figure 1.30. Structural diagram of a system

Answer:

$$W(p) = \frac{Y(p)}{U(p)} = \frac{W_1 W_2 W_3}{1 - W_1 W_2 H_1 + W_2 W_3 H_2 + W_1 W_2 W_3}$$

10. A supersonic aircraft with a wing control system has a structural diagram, as depicted in Figure 1.31. Two gyroscopes are used for angle and speed feedback. The moment M acts on the vertical axis of the aircraft by tilting the two auxiliary wings in opposite directions. Find the closed system transfer function in terms of error and in terms of restraining moment M_c .

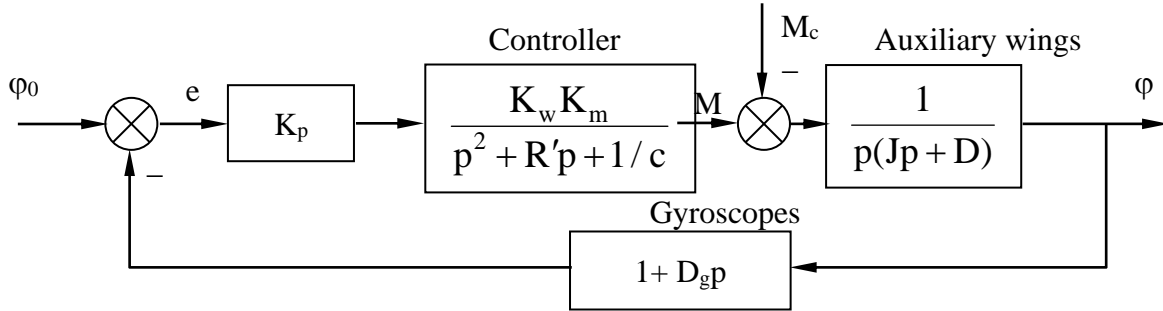


Figure 1.31. Wing control system of supersonic aircraft

Answer:

$$W_k(p) = \frac{\varphi(p)}{\varphi_0(p)} = \frac{K_p K_w K_m}{p(Jp + D)(Ip^2 + R'p + 1/C) + K_p K_w K_m (1 + D_g p)}$$

$$W_E(p) = \frac{E(p)}{\varphi_0(p)} = \frac{p(Jp + D)(Ip^2 + R'p + 1/C)}{p(Jp + D)(Ip^2 + R'p + 1/C) + K_p K_w K_m (1 + D_g p)}$$

$$W_F(p) = \frac{\varphi(p)}{M_c(p)} = \frac{-(1 + D_g p)(Ip^2 + R'p + 1/C)}{p(Jp + D)(Ip^2 + R'p + 1/C) + K_p K_w K_m (1 + D_g p)}$$

11. Determine the transfer function of the system with the structural diagram shown in Figure 1.32.

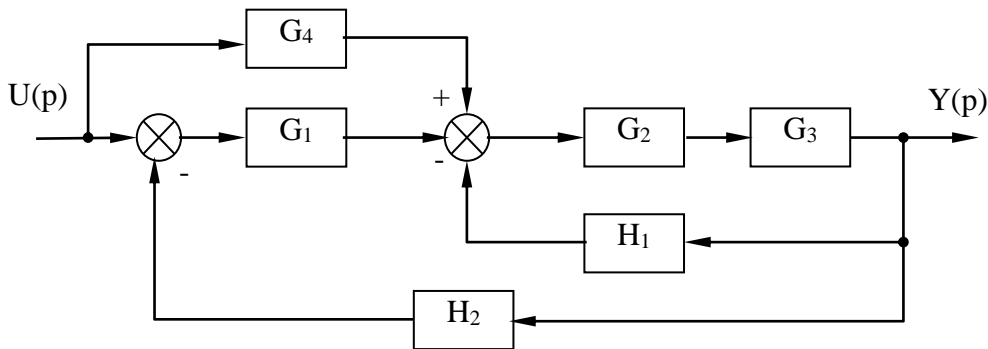


Figure 1.32. Structural diagram of a system

Answer :

$$W(p) = \frac{Y(p)}{U(p)} = \frac{G_1 G_2 G_3 + G_2 G_3 G_4}{1 + H_1 G_2 G_3 + H_2 G_1 G_2 G_3}$$

12. For the autopilot system of a supersonic aircraft as shown in Figure 1.33.

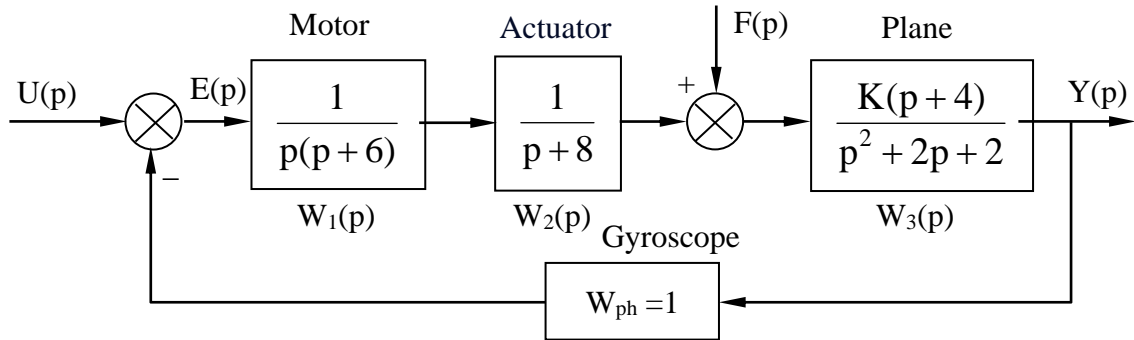


Figure 1.33. Autopilot system of supersonic aircraft

Find the transfer function for open system, closed system, error and noise.

13. Give a shortened structure diagram of the rocket self-guidance control system along the flight plane as depicted in Figure 1.34. Find the transfer function for the open system, the closed system, and the deviation of the system according to the given parameters.

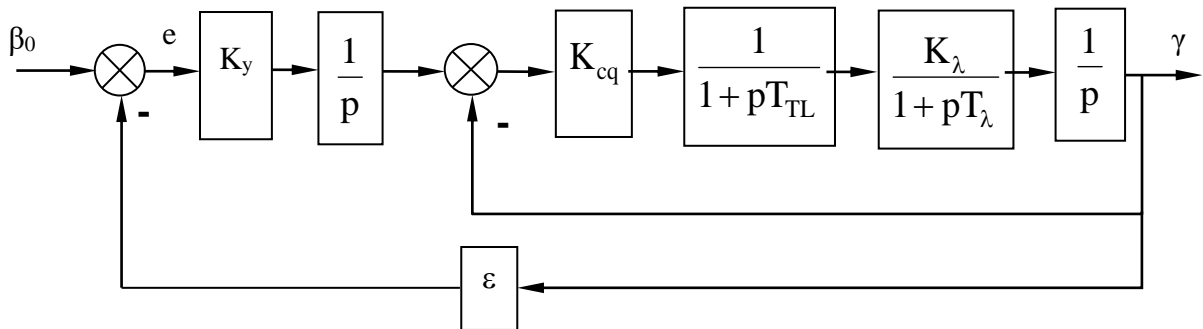


Figure 1.34. The control system automatically guides the missile along the throw plane

14. Survey the telephone network transmitting signals, including noise (interference) as described in Figure 1.35. In the feedback path there is a filter $H(p)$ to keep the closed system stable. Determine the closed system transfer function and the signal $Y(p)$ at the receiver according to each noise $F_1(p)$, $F_2(p)$. Know $f_1(t) = f_2(t) = \delta(t)$.

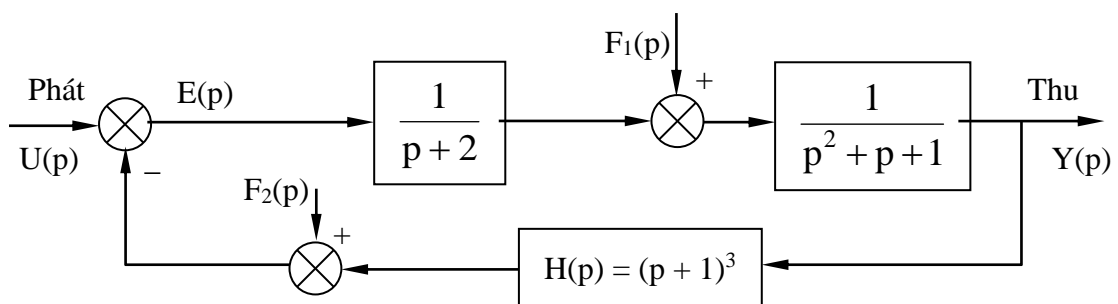


Figure 1.35. The telephone network has transmission interference

Conclusion of chapter 1

Chapter one presented methods to describe linear automatic control systems. Depending on the system and control problem to be solved, we choose the appropriate description method. If the problem is to analyze a system with one input, one output and the relationship between input and output can be expressed by a constant coefficient differential equation, then the transfer function method can be chosen to describe the system. system.

Chapter 1 review questions

Question 1: Analyze the basic components and signals of the automatic control system? Control principles?

Question 2: Definition and properties of Laplace transform?

Question 3: What is the method to describe the control system using differential equations and transfer functions?

Question 4: What are the basic rules for transforming structural diagrams?

Question 5: What are the types of communication functions of structural diagrams?

Chapter 2

DYNAMIC CHARACTERISTICS OF AUTOMATIC CONTROL SYSTEM

The system's dynamic characteristics describe the change in the output signal over time when there is an input. In reality, automatic control systems are very diverse, but systems described by mathematical models of the same form will have similar dynamic characteristics. To investigate the dynamic characteristics of the system, the input signal is often chosen as a basic signal such as a unit step function, a unit impulse function, or a harmonic function. Depending on the type of input signal, the resulting dynamic characteristics are either time characteristics or frequency characteristics.

2.1. BASIC CHARACTERISTICS OF AUTOMATIC CONTROL SYSTEM

2.1.1. Typical signals

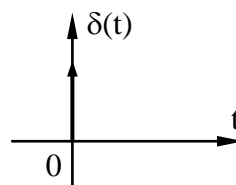
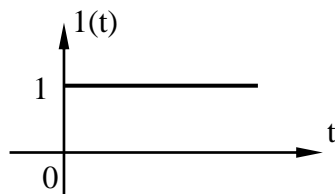
Typical signals are predetermined signals, typical in practice, characteristic of different impacts, reflecting the most severe nature and most likely working mode of the system.

Typical signals are easily generated experimentally using technical equipment. Simple in mathematics, it is easy to investigate the system using mathematics. If the system satisfies the working requirements for typical signals, it will also satisfy the working requirements for other common signals.

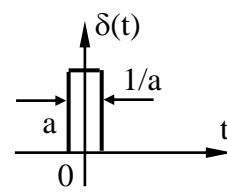
a) Unit ladder signal

Mathematical expression of the unit ladder signal:

$$1(t) = \begin{cases} 0 & \text{when } t < 0 \\ 1 & \text{when } t \geq 0 \end{cases} \quad (2.1)$$



(a)



(b)

Figure 2.1. Signal graph 1(t)

Figure 2.2. Graph of unit pulse signal

The graph of the unit step signal is depicted in Figure 2.1, this signal characterizes the sudden change of external influences (for example: Load, control rotation angle...).

b) Unit pulse signal

Mathematical expression of the unit pulse signal (Dirac pulse):

$$\delta(t) = \begin{cases} 0 & \text{when } t \neq 0 \\ \infty & \text{when } t = 0 \end{cases} \quad (2.2)$$

The graph of the unit pulse signal is depicted in Figure 2.2 ((a) is the theoretical signal graph, (b) is the actual signal graph), this signal characterizes the effects that occur, occurring in a short period of time (for example, the impact of airflow on the autopilot system, impulse loads in electrical and electromechanical systems...).

There is a relationship between the unit step function and the unit impulse function as follows:

$$\delta(t) = \frac{d1(t)}{dt} \quad (2.3)$$

Area of pulse:

$$S = \int_0^t \delta(t) dt = \int_0^t \frac{d1(t)}{dt} dt = 1 \quad (2.4)$$

The function $\delta(t)$ has the following properties:

$$\int_{-\infty}^{\infty} \delta(t) dt = 1 \quad (2.5)$$

In reality, the unit pulse has a certain height and width. To have a width of one, the height must be the inverse of the width. The smaller the width, the closer the actual function is to the theoretical function.

c) *Harmonic signal*

Mathematical expression of harmonic signal:

$$u(t) = U_m \cdot \sin \omega t \quad \text{or} \quad u(t) = U_m \cdot \cos \omega t \quad (2.6)$$

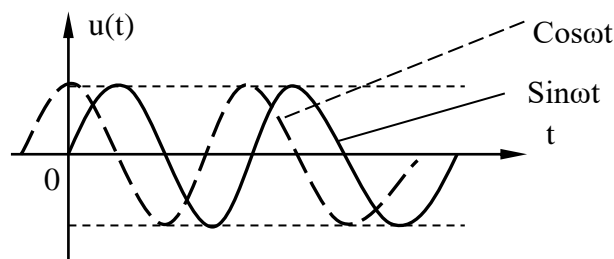


Figure 2.3. Describe harmonic signals

When the input to the system is a harmonic signal, the output is a harmonic signal, then we can determine the frequency characteristics of the system.

d) *Represent any signal through a typical signal*

Signals of any type can be represented as either a $1(t)$ signal or a $\delta(t)$ signal. To express $u(t)$ in terms of $1(t)$ and $\delta(t)$ we decompose $u(t)$ by the Duamen integral.

Represent any signal $u(t)$ through $1(t)$:

$$u(t) = \lim_{\alpha \rightarrow 0} u(\alpha) \cdot 1(t) + \int_0^t \frac{du(\tau)}{d\tau} \cdot 1(t - \tau) d\tau \quad (2.7)$$

To represent any signal $u(t)$ through $\delta(t)$, we implement $u(t)$ according to the formula:

$$u(t) = \lim_{\alpha \rightarrow 0} \int_{\alpha}^{t+\alpha} u(\tau) \cdot \delta(t - \tau) d\tau \quad (2.8)$$

If $x(t)$ is definite and continuous for all values of t then:

$$u(t) = \int_{-\infty}^{\infty} u(\tau) \cdot \delta(t - \tau) d\tau \quad (2.9)$$

2.1.2. Basic characteristics

a) Time characteristics

The timing characteristic is the response of the system when the input is signal $1(t)$ or $\delta(t)$.

- Excessive jaw

The transient function $h(t)$ is the response of the system when the impact is a unit step signal $1(t)$.

If we know the transfer function of the system, we can determine $h(t)$ using Laplace's inverse transform formula (Laplace image of signal $1(t)$ is $1/p$):

$$h(t) = L^{-1} \{ U(p) \cdot W(p) \} = L^{-1} \left\{ \frac{1}{p} \cdot W(p) \right\} \quad (2.10)$$

Where $W(p)$ is the transfer function of the system.

The description of the transition function on the graph is called the transition characteristic. Transient characteristics include two types: Non-oscillating transient characteristics are described in Figure 2.4a and oscillatory transient characteristics are described in Figure 2.4b.

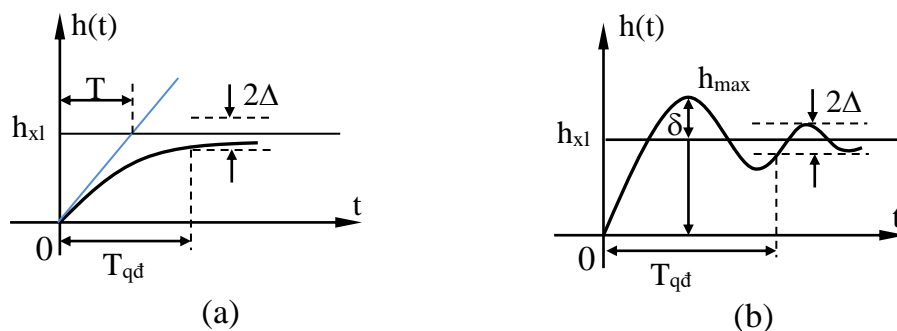


Figure 2.4. Describe transient characteristics in the system

For systems with non-oscillating transient characteristics (first-order systems), quality assessment criteria often include:

The time constant T is the time interval between the vertical axis and the intersection of line h_{xl} with the tangent of line $h(t)$ at the origin.

The transition time T_{qd} is the time period from the origin to the moment the transition process reaches a set value with static error Δ .

Static error Δ (or ε) is the deviation of the output quantity $h(t)$ from its steady value at a time equal to the transition time.

For systems with fluctuating transient characteristics, quality assessment criteria often include:

Transient time T_{qd} (defined as non-oscillating transient characteristic).

Overcorrection δ is the ratio of the maximum deviation of $h(t)$ from its set value divided by the set value in percent.

$$\delta = \frac{h_{\max} - h_{xl}}{h_{xl}} 100\% \quad (2.11)$$

The fluctuation n is the number of fluctuations in the transition period T_{qd} .

$$n = \frac{T_{qd}}{T_0} \quad (2.12)$$

Where T_0 is the oscillation period.

Static error Δ (defined as a non-oscillating transient characteristic).

- Weight function

The weight function $g(t)$ is the system's response when the impact is a unit pulse signal $\delta(t)$.

If we know the transfer function of the system, we can determine $g(t)$ using Laplace's inverse transform formula (Laplace image of signal $\delta(t)$ is 1):

$$g(t) = L^{-1} \{W(p).1\} = L^{-1} \{W(p)\} \quad (2.13)$$

Relationship between $h(t)$ and $g(t)$:

$$h(t) = \int_0^t g(t) dt \quad \text{good} \quad g(t) = \frac{dh(t)}{dt} \quad (2.14)$$

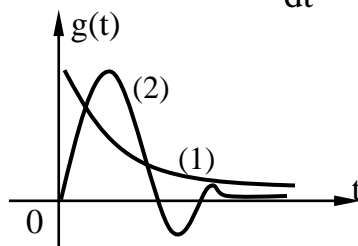


Figure 2.5. Describe the pulse characteristics of the system

The description of the weight function on the graph is called the impulse characteristic. Similar to the transition function, the weight function also has two types: non-oscillating characteristics (line 1) and fluctuating characteristics (line 2) as described in figure 2.5. From the relationship (2.14), we see that it is possible to use both the transition function and the weight function to investigate the dynamic characteristics of the system, but in reality people often use the transition function when studying the system. Save the automatic control system.

b) Frequency characteristics

- Frequency transfer function

Definition: The frequency transfer function is the ratio between the output and input of the system (or element) in a steady state. When the input changes according to the harmonic law $u(t) = U_m \cdot \sin \omega t$, the output also changes according to the law. air conditioning $y(t) = Y_m \cdot \sin(\omega t + \phi)$.

Suppose an automatic control system is described by the following differential equation:

$$a_0 \frac{d^n y(t)}{dt^n} + a_1 \frac{d^{n-1} y(t)}{dt^{n-1}} + \dots + a_n y(t) = b_0 \frac{d^m u(t)}{dt^m} + b_1 \frac{d^{m-1} u(t)}{dt^{m-1}} + \dots + b_m u(t) \quad (2.15)$$

In which: $u(t)$, $y(t)$ are the input quantity and output quantity,

m , n are the highest level of input and output,

a_i , b_j are the coefficients of the differential equation ($i = 0, \dots, n$; $j = 0, \dots, m$).

Expressed as a complex of input quantities, output quantities and derivatives of all levels, we get:

$$\left\{ \begin{array}{ll} u(t) = U_m \cdot \sin \omega t & \Rightarrow U_m \cdot e^{j\omega t} \\ \frac{d^m u(t)}{dt^m} = U_m \cdot \omega^m \cdot \sin(\omega t + m \cdot \frac{\pi}{2}) & \Rightarrow U_m \cdot (j\omega)^m \cdot e^{j\omega t} \\ y(t) = Y_m \cdot \sin(\omega t + \phi) & \Rightarrow Y_m \cdot e^{j(\omega t + \phi)} \\ \frac{d^n y(t)}{dt^n} = Y_m \cdot \omega^n \cdot \sin(\omega t + \phi + n \cdot \frac{\pi}{2}) & \Rightarrow Y_m \cdot (j\omega)^n \cdot e^{j(\omega t + \phi)} \end{array} \right. \quad (2.16)$$

At this point, the corresponding other equation representation of (2.15) is:

$$\left[\sum_{i=0}^n a_i \cdot (j\omega)^{n-i} \right] \cdot Y_m \cdot e^{j(\omega t + \phi)} = \left[\sum_{j=0}^m b_j \cdot (j\omega)^{m-j} \right] \cdot U_m \cdot e^{j\omega t} \quad (2.17)$$

According to the definition: The frequency transfer function of a system is the ratio of the complex amplitude of the output quantity to the complex amplitude of the input quantity, we have:

$$W(j\omega) = \frac{Y_n}{U_m} \cdot e^{j\phi} = \frac{b_0(j\omega)^m + b_1(j\omega)^{m-1} + \dots + b_{m-1}(j\omega) + b_m}{a_0(j\omega)^n + a_1(j\omega)^{n-1} + \dots + a_{n-1}(j\omega) + a_n} \quad (2.18)$$

The expression (2.18) is called the frequency transfer function of the system (or element).

Comparing the frequency transfer function with the transfer function expression (1.22), we can determine $W(j\omega)$ by replacing $p = j\omega$, then:

$$W(j\omega) = W(p)|_{p=j\omega} \quad (2.19)$$

We can easily see that the frequency transfer function $W(j\omega)$ is a complex number, depending on the frequency. Through $W(j\omega)$, it is possible to determine the gain coefficient, phase difference angle for the alternating signal and determine the equation of the output signal during the establishment process.

- Fundamental frequency characteristics
- + Frequency phase amplitude characteristics (Nyquist chart)

The frequency phase amplitude characteristic is the trajectory of the frequency transfer function $W(j\omega)$ on the complex plane when ω varying from $0 \div \infty$. The characteristic $W(j\omega)$ is a vector whose length and angle vary with frequency. If ω the variation is from 0 to ∞ the top point of the vector $W(j\omega)$ will trace a curve as shown in Figure 2.6.

This curve is called the velocity phase-frequency characteristic or Nyquist plot. Each point on the graph corresponds to both the real value and the imaginary value of the vector $W(j\omega)$ at a certain frequency. The characteristic $W(j\omega)$ is obtained from the frequency transfer function where the order in the numerator is always smaller than the order in the sample, so when ω approaches ∞ then $|W(j\omega)| \rightarrow 0$.

The complex function $W(j\omega)$ can be written algebraically:

$$W(j\omega) = P(\omega) + jQ(\omega) \quad (2.20)$$

Or write in amplitude - phase form:

$$W(j\omega) = A(\omega)e^{j\arctg\varphi(\omega)} \quad (2.21)$$

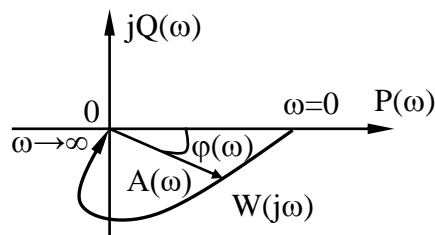


Figure 2.6. Nyquist plot

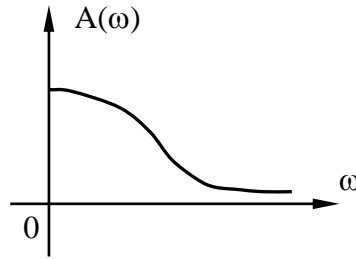


Figure 2.7. Frequency amplitude characteristics

+ Frequency amplitude characteristics

If we know $P(\omega)$ and $Q(\omega)$, we can easily determine $A(\omega)$:

$$A(\omega) = |W(j\omega)| = \sqrt{P^2(\omega) + Q^2(\omega)} \quad (2.22)$$

For ω variation from 0 to ∞ , then $A(\omega)$ will trace a curve of the form shown in Figure 2.7.

Frequency phase characteristics

$$\varphi(\omega) = \text{arctg} \frac{Q(\omega)}{P(\omega)} \quad (2.23)$$

Frequency phase characteristics are often measured in rads on the vertical axis, and in rad/s or dec units on the horizontal axis.

When ω varying from 0 to ∞ we can draw the phase-frequency characteristic as shown in Figure 2.8.

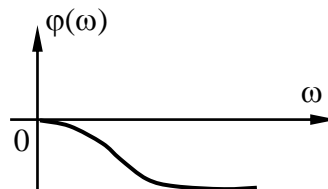


Figure 2.8. Frequency phase characteristics

Frequency amplitude logarithm characteristics

To simplify graphing and facilitate the process of studying the dynamic properties of the system, the logarithmic characteristic of frequency amplitude is used. The logarithm property helps convert products and quotients into algebraic sums. To the extent of allowable error, the terms in the algebraic sum can be considered to depend linearly on frequency, meaning that the curve form $A(\omega)$ can be transformed into an approximate straight line form.

So if we know $A(\omega)$, we can determine the logarithmic characteristic of frequency amplitude in dB units as

$$L(\omega) = 20 \lg A(\omega) \quad (2.24)$$

The way to represent $L(\omega)$ is described in Figure 2.9:

The horizontal axis is divided by $\lg \omega$, directly record the value of ω but divided by decimal logarithm.

The horizontal axis can be divided in units of dec (decade), 1 dec is a measure of the distance between two frequencies 10 times apart. For example, $\omega = 1$ corresponds to 0 dec, $\omega = 10$ corresponds to 1 dec:

$$\lg \frac{10\omega}{\omega} = \lg 10 = 1 \text{ [dec]}$$

Thus, the distance between any two frequencies ω_1 and ω_2 is: $\lg \frac{\omega_2}{\omega_1}$ [dec].

The horizontal axis can also be divided by the unit oct (Octavo) corresponding to the logarithm of the frequency increase by 2 times:

$$1 \text{ oct} = \lg 2 \frac{\omega_1}{\omega_1} = \lg 2 \approx 0,3.$$

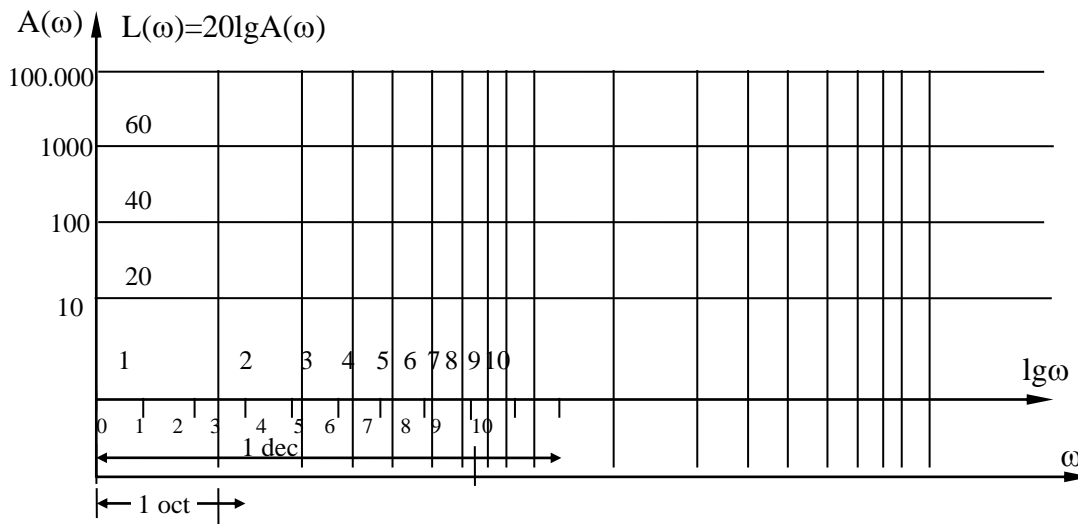


Figure 2.9. Represents the logarithmic characteristic of frequency amplitude

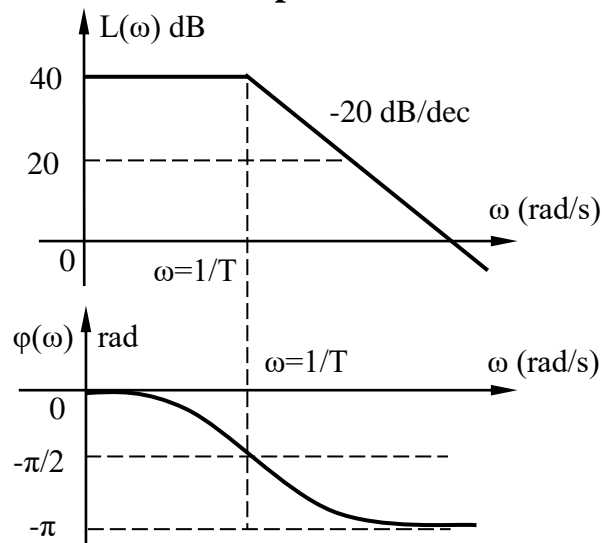


Figure 2.10. Amplitude and phase Bode plot

In addition, we can express the slope of the curve $L(\omega)$ according to the following formula with the unit dB/dec:

$$\text{tg}\alpha = \frac{L(\omega_2) - L(\omega_1)}{\lg(\omega_2) - \lg(\omega_1)} \quad [\text{dB} / \text{dec}] \quad (2.25)$$

In fact, people often represent the logarithmic characteristics of frequency amplitude and frequency phase characteristics on the same coordinate system with the horizontal axis having the same units and the vertical axis coinciding but with different units. Then called the Bode plot, it includes the amplitude Bode plot $L(\omega)$ and the phase Bode plot $\varphi(\omega)$, as depicted in Figure 2.10.

To simplify when drawing Bode plots, people often replace the $L(\omega)$ curve with asymptotic lines with ΔL error smaller than the allowed level of $\pm 3\text{dB}$.

2.2. TYPICAL KINEMATIC STAGES

2.2.1. Overview of typical kinematic stages

a) Concept

In an automatic control system, although the system's elements have the same function, their structure and dynamic properties can be completely different. The dynamic properties of particles are characterized by their dynamic equations, such as differential equations, transfer functions, or equations of state. Therefore, if we examine each group of elements with the same type of kinematic behavior, it will allow us to examine the dynamic properties of the general and convenient system, this can be illustrated with the following simple example. .

Suppose there are two elements described in Figure 2.9.

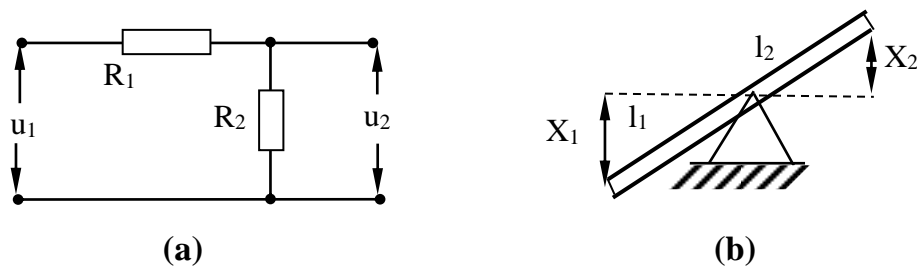


Figure 2.11. Electrical elements and mechanical elements have the same kinematic equation form

Figure 2.9a is the element that will transform the voltage from u_1 to u_2 with the kinetic equation

$$u_2 = \frac{R_2}{R_1 + R_2} \cdot u_1 = K \cdot u_1 \quad (2.26)$$

Figure 2.9b is the element that will transform the displacement from x_1 to x_2 with the kinematic equation

$$x_2 = \frac{l_2}{l_1} \cdot x_1 = K \cdot x_1 \quad (2.27)$$

The above two elements have completely different structures and physical processes, but the description of their dynamic process is completely the same (compare the two expressions (2.26) and (2.27)).

Concept:

In automatic control theory, elements with similar differential equations are classified into a type called kinematics.

Typical kinematic stages are kinematic stages whose differential equations describing the dynamic process are of order less than or equal to two.

b) Classification

Based on the characteristics of differential equations, typical kinematics stages can be divided into three types: integral stages, differential stages and integral stages.

- Suture the original jaw

The primitive stage is a typical kinematic stage in which the output quantity $y(t)$ is proportional to the input quantity $u(t)$.

Typical primitive links are: Amplifying link (proportional link), inertia link, oscillation link, hysteresis link and unstable link.

- Differential stitching

The differential step is a typical dynamic step that in the mode of setting the output quantity $y(t)$ is proportional to the input quantity derivative $du(t)/dt$.

- Integral stage

The integration step is a typical dynamic step in which the output quantity $y(t)$ is set proportional to the input quantity integration $\int u(t)dt$.

2.2.2. Elemental stages

a) Amplification stage

- Differential equations

The amplification stage is a typical dynamic stage where the differential equation has the form:

$$y(t) = Ku(t) \quad (2.28)$$

In which: K is the transmission coefficient of the stage if the output and input quantities do not have the same dimension, is the amplification coefficient if the output and input quantities have the same dimension.

Examples of elements equivalent to the amplification stage: Measurement

elements (interceptor, variable resistor,...); amplifier element (electronic amplifier, semiconductor,...).

- Transfer function

Transforming the Laplace transform on both sides of equation (2.28), we get the following function:

$$W(p) = \frac{Y(p)}{U(p)} = K \quad (2.29)$$

- Time characteristics

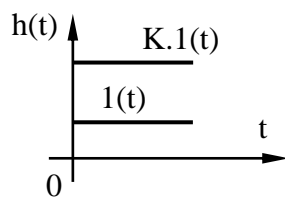
+ Transient function

$$h(t) = L^{-1} \left\{ W(p) \cdot \frac{1}{p} \right\} = L^{-1} \left\{ K \cdot \frac{1}{p} \right\} = K \cdot 1(t) \quad (2.30)$$

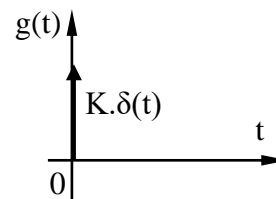
+ Weight function

$$g(t) = \frac{dh(t)}{dt} = K \cdot \frac{d1(t)}{dt} = K \cdot \delta(t) \quad (2.31)$$

Description of the time characteristics of the amplification stage is presented in Figure 2.12.



(a) Đặc tính quá độ



(b) Đặc tính xung

Figure 2.12. Time characteristics of the amplification stage

- Frequency characteristics

Frequency phase amplitude characteristics:

$$W(j\omega) = W(p) \Big|_{p=j\omega} = K \quad (2.32)$$

Frequency amplitude characteristics:

$$A(\omega) = K \quad (2.33)$$

Frequency amplitude logarithmic characteristic:

$$L(\omega) = 20 \lg K \quad (2.34)$$

Frequency phase characteristics:

$$\varphi(\omega) = 0 \quad (2.35)$$

Description of the frequency characteristics of the amplifier stage is presented in Figure 2.13.

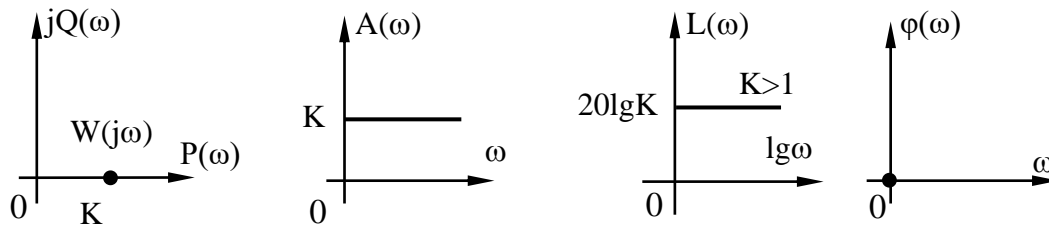


Figure 2.13. Frequency characteristics of the amplifier stage

From the characteristics of the amplification stage, we see that the amplification stage is only responsible for amplifying the signal K times; The input and output signals of the amplifier stage are in phase with each other.

b) Inertia stage

- Differential equations

The inertial step (first order) is a typical kinematic step whose differential equation has the form:

$$T \frac{dy(t)}{dt} + y(t) = Ku(t) \quad (2.36)$$

In which: K is the transmission coefficient and T is the time constant of the inertial link.

Examples of elements equivalent to inertial stages: Magnetic amplifier, DC generator, RC and LR circuits, 2-phase and 3-phase asynchronous motors if the output is angular speed, heat furnace, spring - damper system, ...

- Transfer function

Laplace transform both sides of equation (2.36), setting the ratio of output quantity and input quantity, we obtain the transfer function:

$$W(p) = \frac{Y(p)}{U(p)} = \frac{K}{Tp + 1} \quad (2.37)$$

- Time characteristics

+ Transient function

$$h(t) = L^{-1} \left[\frac{K}{Tp + 1} \cdot \frac{1}{p} \right] = K \cdot \left(1 - e^{-\frac{t}{T}} \right) \quad (2.38)$$

+ Weight function

$$g(t) = \frac{dh(t)}{dt} = \frac{K}{T} \cdot e^{-\frac{t}{T}} \quad (2.39)$$

Description of the time characteristics of the inertial link is presented in Figure 2.14.

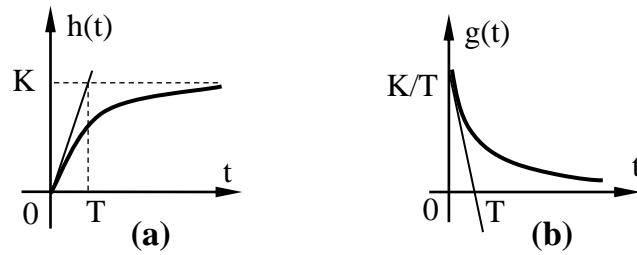


Figure 2.14. Time characteristics of the inertial link

- Frequency characteristics

+ Frequency phase amplitude characteristics

$$W(j\omega) = W(p)|_{p=j\omega} = \frac{K}{Tj\omega + 1} = \frac{K}{1 + \omega^2 T^2} + j \cdot \frac{-K\omega T}{1 + \omega^2 T^2} \quad (2.40)$$

+ Frequency amplitude characteristics

$$A(\omega) = \sqrt{P^2(\omega) + Q^2(\omega)} = \frac{K}{\sqrt{T^2 \omega^2 + 1}} \quad (2.41)$$

To approximate the Nyquist plot and frequency amplitude characteristics, we let ω the variation from $0 \div \infty$, then calculate some $P(\omega)$ and $Q(\omega)$ values as shown in table 2.1.

The Nyquist diagram of the inertial link is the lower half of the circle with center $(K/2, j0)$, radius $K/2$.

+ Frequency amplitude logarithmic characteristics

$$L(\omega) = 20 \lg A(\omega) = 20 \lg K - 20 \lg \sqrt{1 + \omega^2 T^2} \quad (2.42)$$

To draw the amplitude Bode plot, we let ω the variation from 0 to ∞ , determine the corresponding $L(\omega)$ values and then show it on the graph.

Table 2.1. Value of frequency characteristics according to ω

ω	0	...	1/T	...	∞
$P(\omega)$	K	...	K/2	...	0
$Q(\omega)$	0	...	-K/2	...	0
$A(\omega)$	K	...	$K/\sqrt{2}$...	0
$\varphi(\omega)$	0	...	-45°	...	-90°

Steps to draw an approximate amplitude Bode chart:

Step 1: Draw two asymptotes

When $0 < \omega \leq 1/T$: Because $\lim_{\omega \rightarrow 0} 20 \lg \sqrt{T^2 \omega^2 + 1} \approx 20 \lg 1 = 0$, should:

$$L_1(\omega) = 20\lg K$$

When $\omega \geq 1/T$: Because $\lim_{\omega \rightarrow \infty} 20\lg \sqrt{T^2\omega^2 + 1} \approx 20\lg \omega T$:

$$L_2(\omega) = 20\lg K - 20\lg \omega T$$

Thus we get:

$$L(\omega) = \begin{cases} L_1(\omega) = 20\lg K & \text{khi } 0 < \omega \leq 1/T \\ L_2(\omega) = 20\lg K - 20\lg \omega T & \text{khi } \omega > 1/T \end{cases} \quad (2.43)$$

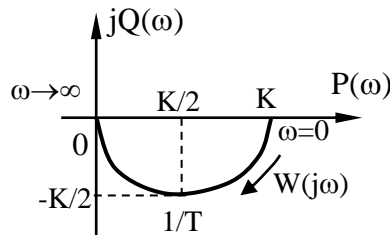


Figure 2.15. Nyquist plot

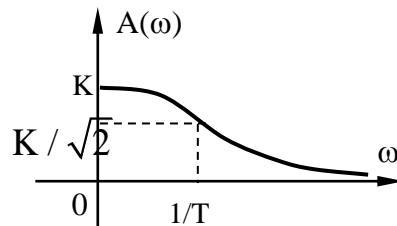


Figure 2.16. Frequency amplitude characteristics

Step 2 : Determine the slope of the asymptotes

The slope of line $L_1(\omega)$ is: 0 [dB/dec], because it is parallel to the horizontal axis.

The slope of line $L_2(\omega)$ is:

According to formula (2.25), choose $\omega_2 = 10\omega_1$, then the slope of line $L_2(\omega)$:

$$\text{tg}\alpha = \frac{L_2(\omega_2) - L_2(\omega_1)}{\lg(\omega_2) - \lg(\omega_1)} = \frac{20\lg K - 20\lg 10\omega_1 T - 20\lg K + 20\lg \omega_1 T}{\lg 10\omega_1 - \lg \omega_1} = -20[\text{dB} / \text{dec}]$$

Step 3 : Determine the folding frequency ω_G

$L_1(\omega)$ and $L_2(\omega)$ intersect at point G, the frequency corresponding to that point is called the folding frequency ω_G . At point G we have $L_1(\omega_G) = L_2(\omega_G)$, that is, $20\lg K = 20\lg K - 20\lg \omega_G T$, so $\omega_G = 1/T$.

Step 4: Determine the maximum amplitude error $\Delta L(\omega_G)$

The difference $\Delta L(\omega_G)$ between the exact line and the asymptote at point G is:

$$\Delta L(\omega_g) = L_1(\omega) - L_2(\omega) = 20\lg K - 20\lg K + 20\lg \sqrt{T^2 \left(\frac{1}{T}\right)^2 + 1} = 20\lg \sqrt{2} = 3[\text{dB}]$$

From the steps above, we can draw the amplitude Bode plot of the inertial link as depicted in Figure 2.17 with the exact line following the asymptote.

+ Frequency phase characteristics

$$\varphi(\omega) = \arctg \frac{Q(\omega)}{P(\omega)} = -\arctg \omega T \quad (2.44)$$

According to table 2.1, for ω varying from $0 \div \infty$, we can draw the frequency phase characteristics as described in figure 2.18.

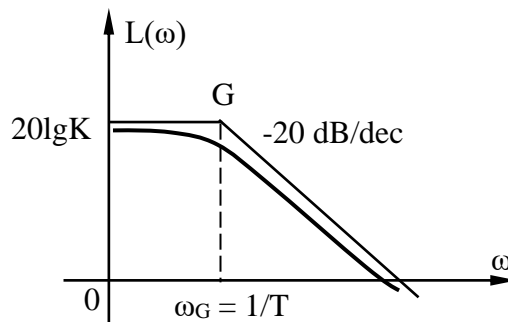


Figure 2.17. Amplitude Bode plot

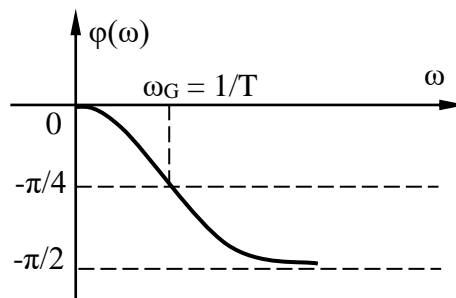


Figure 2.18. Frequency phase characteristics

Through examining the inertial phase, we can draw a number of observations:

From the transition function $h(t)$ of the inertial link, we see that the inertial link does not immediately reach the K value but gradually approaches the K value according to the exponential law (so the inertial link is also called a non-cyclic link). Thus, the process of accumulating energy and releasing energy does not occur at the same time, causing the phenomenon of inertia.

From the weight function $g(t)$ of the inertial link, we see that when the transition function reaches a set value, the weight function will decrease to zero, meaning that at this time the inertial link is released from inertia.

Analyzing the timing characteristics of the inertial link, we see that the smaller the time constant T , the faster the output response reaches the established value.

From the frequency amplitude characteristic $A(\omega)$, we see that the inertial stage cannot work with high-frequency signals (the $A(\omega)$ characteristic is like a low-pass filter).

From the frequency phase characteristics $\varphi(\omega)$, we see that the output signal of the inertial link is always slower in phase than the input signal by an angle varying from $0 \div \pi/2$ according to the change of ω , meaning that the inertial link has a slow impact.

c) Oscillating stage

- Differential equations

The oscillation stage is a typical kinematic stage whose differential equation has the form:

$$T^2 \frac{d^2y(t)}{dt^2} + 2\xi T \frac{dy(t)}{dt} + y(t) = K.u(t) \quad (2.45)$$

In which: T is the time constant; K is the transmission coefficient; ξ is the damping coefficient ($0 < \xi < 1$).

Examples of elements equivalent to oscillation stages: RLC circuit (example 1.2.1); In an independently excited DC motor, the input quantity is the armature voltage, the output quantity is the rotational angular speed; elastic mechanical system; gyroscope; ...

- Transfer function

Laplace transform both sides of equation (2.45), setting the ratio of output quantity and input quantity, we obtain the transfer function:

$$W(p) = \frac{Y(p)}{U(p)} = \frac{K}{T^2 p^2 + 2\xi T p + 1} \quad (2.46)$$

We can express (2.46) as:

$$W(p) = \frac{K\omega_n^2}{p^2 + 2\xi\omega_n p + \omega_n^2} \quad (2.47)$$

Where: $\omega_n = \frac{1}{T}$ is the natural oscillation frequency.

- Time characteristics

+ Transient function

$$h(t) = L^{-1} \left\{ \frac{K\omega_n^2}{p^2 + 2\xi\omega_n p + \omega_n^2} \cdot \frac{1}{p} \right\} \quad (2.48)$$

Consider the cases of changing the damping coefficient ξ .

If $\xi = 0$, then

$$h(t) = L^{-1} \left\{ \frac{K\omega_n^2}{p(p^2 + \omega_n^2)} \right\} = K \left(\frac{1}{p} - \frac{p}{p^2 + \omega_n^2} \right)$$

Looking up the Laplace transform table, we find:

$$h(t) = K(1 - \cos\omega_n t) \quad (2.49)$$

Thus, in the case $\xi = 0$, the transient response is an undamped oscillation form with natural oscillation frequency ω_n .

If $0 < \xi < 1$, polynomial denominator: $H_2(p) = p(p^2 + 2\xi\omega_n p + \omega_n^2)$, then the (extreme) solution of the equation $H_2(p) = 0$ is (Figure 2.19):

$$p_1 = 0; p_{2,3} = -\omega_n \xi \pm j\omega_n \sqrt{1 - \xi^2} = -\alpha \pm j\omega_d$$

Applying Heviside's formula to expression (2.48) and transforming, we find:

$$h(t) = K \left[1 - e^{-\alpha t} \left(\cos \omega_d t + \frac{\alpha}{\omega_d} \sin \omega_d t \right) \right] \quad (2.50)$$

$$\text{In there: } \alpha = \xi \cdot \omega_n; \omega_n = \frac{1}{T}; \omega_d = \omega_n \cdot \sqrt{1 - \xi^2}$$

Can be rewritten (2.50) as:

$$h(t) = K \left[1 - \frac{e^{-\alpha t}}{\sqrt{1 - \xi^2}} \sin(\omega_d t + \theta) \right] \quad (2.51)$$

$$\text{In which: } \theta = \text{arctg} \frac{\sqrt{1 - \xi^2}}{\xi} \text{ or } \cos \theta = \xi.$$

Thus, in the case $0 < \xi < 1$: The transient response is an oscillation form with amplitude decreasing according to an exponential law. The larger ξ is (the closer the pole is to the real axis), the faster the oscillation decreases.

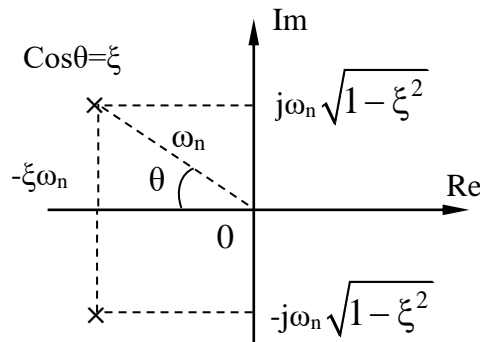


Figure 2.19. The pole of the link vibrates

If $\xi = 1$, the characteristic equation has a double solution: $p_1 = p_2 = -1/T$:

$$H(p) = \frac{K\omega_n^2}{p(p + \omega_n)^2} = K \left[\frac{1}{p} - \frac{1}{p + \omega_n} - \frac{\omega_n^2}{(p + \omega_n)^2} \right]$$

Looking up the Laplace transform table, we get:

$$h(t) = K \left[1 - e^{-\omega_n t} - \omega_n t e^{-\omega_n t} \right] \quad (2.52)$$

If $\xi > 1$, the characteristic equation has two simple solutions:

$$p_1 = -1/T_1 \text{ and } p_2 = -1/T_2. \text{ Then: } H(p) = \frac{K / T_1 T_2}{p(p + 1/T_1)(p + 1/T_2)}$$

Transforming the inverse Laplace, we get:

$$h(t) = K \left[1 - \frac{T_1}{T_1 - T_2} e^{-t/T_1} + \frac{T_2}{T_1 - T_2} e^{-t/T_2} \right] \quad (2.53)$$

With the values of ξ , we can draw the transient characteristics of the oscillating stage as depicted in Figure 2.20.

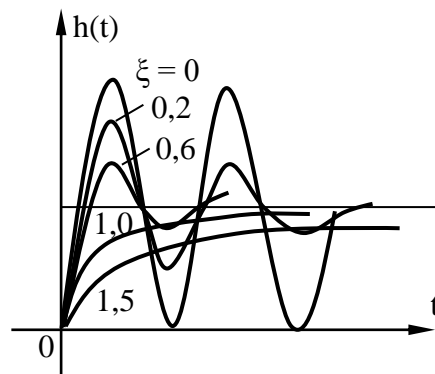


Figure 2.20. Family of transient characteristics

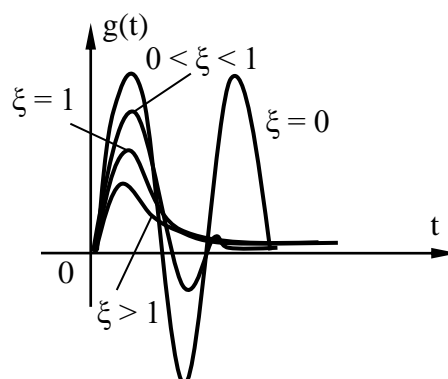


Figure 2.21. They feature pulse transients

+ Weight function

Corresponding to the expressions of the transition function, we find the

weight function:

$$\text{If } \xi = 0: g(t) = \frac{dh(t)}{dt} = K\omega_n \sin \omega_n t$$

$$\text{If } 0 < \xi < 1: g(t) = \frac{dh(t)}{dt} = L^{-1}\{W(p)\} = K \frac{\omega_n^2}{\omega_d} e^{-\alpha t} \sin \omega_d t$$

$$\text{If } \xi = 1: g(t) = \frac{dh(t)}{dt} = \frac{K}{T^2} t e^{-t/T}$$

$$\text{If } \xi > 1: g(t) = \frac{dh(t)}{dt} = \frac{K}{T_1 - T_2} (e^{-t/T_1} - e^{-t/T_2})$$

With the values of ξ , we can draw the pulse characteristics of the oscillating stage as depicted in Figure 2.21.

- Frequency characteristics

+ Frequency phase amplitude characteristics

$$W(j\omega) = W(p)|_{p=j\omega} = \frac{K}{1 - T^2\omega^2 + j2\xi T\omega} \quad (2.54)$$

Multiplying the complex conjugate of the denominator expression by both the numerator and denominator of $W(j\omega)$ we get:

$$W(j\omega) = \frac{K(1 - \omega^2 T^2)}{(1 - \omega^2 T^2)^2 + 4\xi^2 \omega^2 T^2} + j \frac{-K2\xi\omega T}{(1 - \omega^2 T^2)^2 + 4\xi^2 \omega^2 T^2} \quad (2.55)$$

Let ω vary from $0 \div \infty$ with different ξ , we will build a Nyquist diagram of the oscillating stage as depicted in Figure 2.22.

+ Frequency amplitude characteristics

$$A(\omega) = \frac{K}{\sqrt{(1 - T^2\omega^2)^2 + (2\xi T\omega)^2}} \quad (2.56)$$

Find the resonance frequency by solving the equation $\frac{dA(\omega)}{d\omega} = 0$:

$$\omega_{ch} = \frac{1}{T} \cdot \sqrt{1 - 2\xi^2} \quad (2.57)$$

From (2.57) we see: $\omega_{ch}^2 \cdot T^2 = 1 - 2\xi^2 \Rightarrow \omega_{ch}^2 \cdot T^2$ always positive. So $A(\omega)$ max when $1 - 2\xi^2 > 0$ hay $\xi < \frac{1}{\sqrt{2}}$.

Substituting the value ω_{ch} into (2.56), we will find the maximum value of

$A(\omega)$ according to ξ :

$$A_{\max}(\omega) = \frac{K}{2\xi\sqrt{1-\xi^2}} \quad (2.58)$$

From here we see that, the smaller ξ is, the larger $A(\omega)$ is. When $\xi \rightarrow 0$ then $A_{\max}(\omega) \rightarrow \infty$ means that the curve $A(\omega)$ is discontinuous as depicted in figure 2.23.

+ Frequency amplitude logarithmic characteristics

$$L(\omega) = 20\lg A(\omega) = 20\lg K - 20\lg \sqrt{(1-\omega^2T^2)^2 + 4\xi^2\omega^2T^2} \quad (2.59)$$

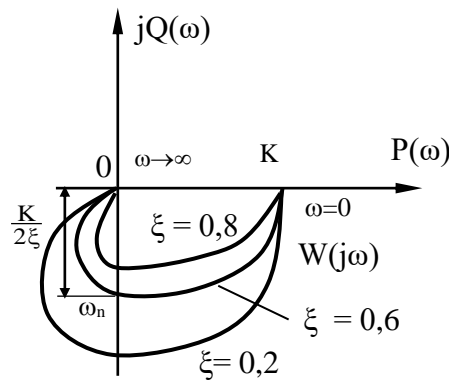
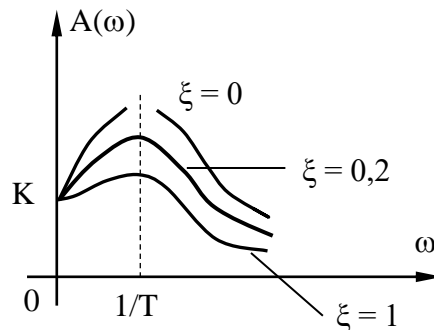


Figure 2.22. Nyquist plot Figure



2.23. Frequency amplitude characteristics

The steps to approximate the amplitude Bode diagram of the oscillating phase are similar to the inertial phase with two asymptote lines:

When $0 < \omega \leq 1/T$:

Because $\lim_{\omega \rightarrow 0} 20\lg \sqrt{(1-\omega^2T^2)^2 + 4\xi^2\omega^2T^2} \approx 20\lg 1 = 0$ should:

$$L_1(\omega) = 20\lg K$$

When $\omega \geq 1/T$:

Because $\lim_{\omega \rightarrow \infty} 20\lg \sqrt{(1-\omega^2T^2)^2 + 4\xi^2\omega^2T^2} \approx 20\lg \omega^2T^2 = 40\lg \omega T$:

$$L_2(\omega) = 20\lg K - 40\lg \omega T$$

Thus we get:

$$L(\omega) = \begin{cases} L_1(\omega) = 20\lg K & \text{khi } 0 < \omega \leq 1/T \\ L_2(\omega) = 20\lg K - 40\lg \omega T & \text{khi } \omega > 1/T \end{cases} \quad (2.60)$$

The only difference here is that $L_2(\omega)$ has a slope of -40dB/dec . With different ξ values (ξ the lower, ω the larger, the $L(\omega)$ characteristic has different shapes).

The logarithmic characteristic of frequency amplitude is described in Figure 2.24.

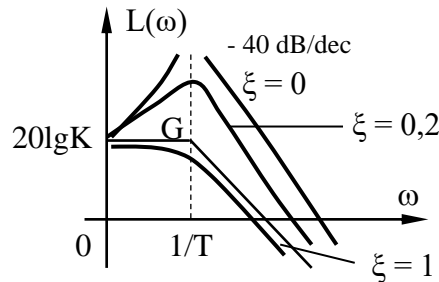


Figure 2.24. Characteristics of frequency amplitude logarithm

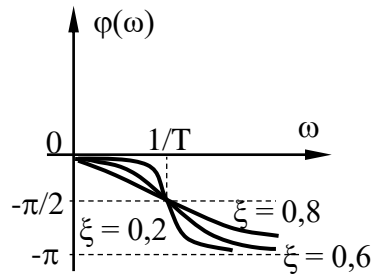


Figure 2.25. Frequency phase characteristics

+ Frequency phase characteristics

$$\varphi(\omega) = -\text{arctg} \frac{2\xi\omega T}{1 - \omega^2 T^2} \quad (2.61)$$

Frequency phase characteristics with different ξ values are described in Figure 2.25.

Through examining the oscillation stage, we can draw a number of observations:

From the transient function $h(t)$ of the oscillating stage, we see that the oscillating stage does not immediately reach the value K but oscillates towards the value K . For the system to oscillate, the system must have a kinetic energy accumulator and a potential accumulator. energy, for example in an RLC circuit, C stores potential energy and L stores kinetic energy.

From the weight function $g(t)$ of the oscillating link, we see that when the transition function reaches a set value, the weight function will decrease to zero, meaning that at this time the oscillating link is released from inertia.

From the frequency amplitude characteristic $A(\omega)$, we see that the oscillator stage cannot work with high-frequency signals ($A(\omega)$ characteristic is like a low-pass filter) and reaches the value $A_{\max}(\omega)$ at ω_{ch} .

From the frequency phase characteristics $\varphi(\omega)$, we see that the output signal of the oscillating link is always slower in phase than the input signal by an angle varying from $0 \div \pi$ according to the change of ω , that is, the oscillating link is reactive. slow movement.

d) Late stitching

- Differential equations

The delay stage is a typical dynamic stage where after a certain period of time the output quantity repeats the input quantity without signal distortion.

The differential equation of the hysteresis has the form:

$$y(t) = u(t - \tau) \tag{2.62}$$

In which: τ is the delay time.

Examples of elements equivalent to delay stages: Conveyor belt, heat pipe if losses are ignored...

- Transfer function

$$W(p) = \frac{Y(p)}{U(p)} = e^{-\tau \cdot p} \tag{2.63}$$

- Time characteristics

+ Transient function

$$h(t) = L^{-1} \left\{ \frac{1}{p} \cdot e^{-\tau \cdot p} \right\} = 1(t - \tau) \tag{2.64}$$

+ Weight function

$$g(t) = L^{-1} \left\{ e^{-\tau \cdot p} \right\} = \delta(t - \tau) \tag{2.65}$$

The transient characteristics and impulse characteristics of the hysteresis link are described in Figures 2.26a and 2.26b, respectively.

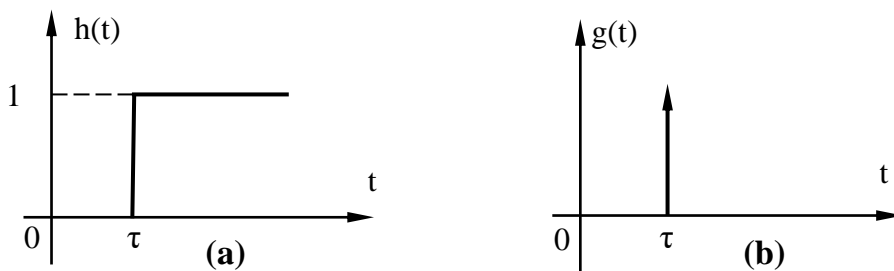


Figure 2.26. Time characteristics of the delay stage

- Frequency characteristics

+ Frequency phase amplitude characteristics

$$W(j\omega) = W(p)\Big|_{p=j\omega} = e^{-j\omega\tau} = -(\cos \omega\tau + j\sin \omega\tau) \quad (2.66)$$

+ Frequency amplitude characteristics

$$A(\omega) = 1 \quad (2.67)$$

The Nyquist plot and frequency amplitude characteristics of the hysteresis are depicted in Figures 2.27a and 2.27b, respectively.

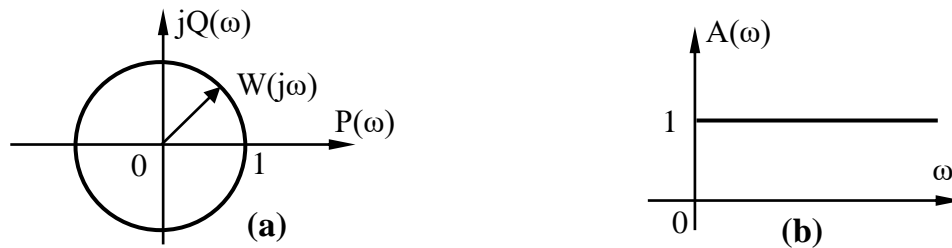


Figure 2.27. Nyquist plot and frequency amplitude characteristics of the hysteresis link

+ Frequency amplitude logarithmic characteristics

$$L(\omega) = 20\lg A(\omega) = 0 \quad (2.68)$$

+ Frequency phase characteristics

$$\varphi(\omega) = -\omega\tau \quad (2.69)$$

The frequency amplitude logarithm and frequency phase characteristics of the delay link are depicted in Figures 2.28a and 2.28b, respectively.



Figure 2.28. Characteristics of logarithm of frequency amplitude and frequency phase of delay link

We see that the delay stage does not change the signal shape, but the delay stage always has an output signal that is out of phase compared to the input signal.

2.2.3. Integral stage

- Differential equations

The integral step is a typical kinematic step where the differential equation has the form:

$$\frac{dy(t)}{dt} = K.u(t) \quad \text{or} \quad y(t) = \frac{1}{T} \int_0^t u(t) dt \quad (2.70)$$

In which:

$T = 1/K$ is the integration time constant

K is the transmission coefficient

Examples of elements equivalent to the integral stage: Water tank - valve system, lead screw - nut set , speed changer box (input is angular speed and output is rotation angle)...

- Transfer function

$$W(p) = \frac{K}{p} = \frac{1}{Tp} \quad (2.71)$$

- Time characteristics

+ Transient function

$$h(t) = L^{-1} \left\{ \frac{K}{p} \cdot \frac{1}{p} \right\} = K.t \quad (2.72)$$

+ Weight function

$$g(t) = \frac{dh(t)}{dt} = K \quad (2.73)$$

The transient characteristics and impulse characteristics of the integrating stage are described in Figures 2.29a and 2.29b, respectively.

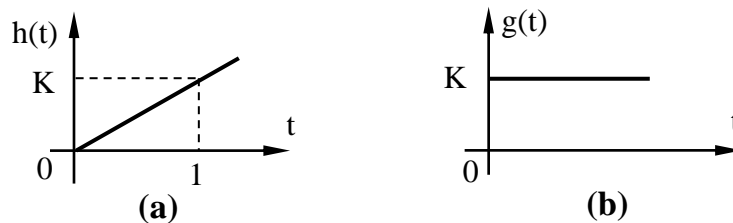


Figure 2.29. Time characteristics of the integration stage

- Frequency characteristics

+ Frequency phase amplitude characteristics

$$W(j\omega) = -j \frac{K}{\omega} = \frac{1}{T\omega} e^{-j\frac{\pi}{2}} \quad (2.74)$$

+ Frequency amplitude characteristics

$$A(\omega) = \frac{K}{\omega} \quad (2.75)$$

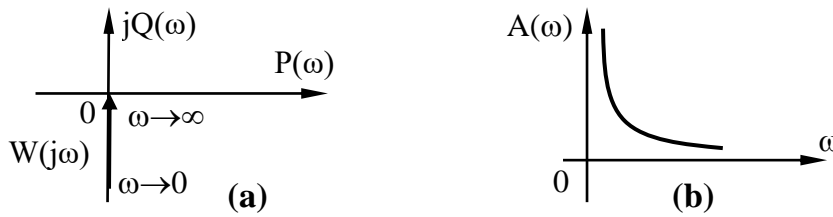


Figure 2.30. Nyquist plot and frequency-amplitude characteristics of the integration stage

+ Frequency amplitude logarithmic characteristics

$$L(\omega) = 20 \lg \frac{K}{\omega} = 20 \lg K - 20 \lg \omega \quad (2.76)$$

The frequency amplitude logarithm characteristic $L(\omega)$ always has the slope: $\text{tg}\alpha = -20\text{dB}/\text{dec}$. At ω_c the characteristic $L(\omega)$ cuts the horizontal axis so $L(\omega_c) = 0$, deducing the cutoff frequency $\omega_c = K$.

+ Frequency phase characteristics

$$\varphi(\omega) = \text{arctg} \left(\frac{-K/\omega}{0} \right) = -\frac{\pi}{2} \quad (2.77)$$

The frequency amplitude logarithm and frequency phase characteristics of the integration stage are described in Figures 2.31a and 2.31b, respectively.

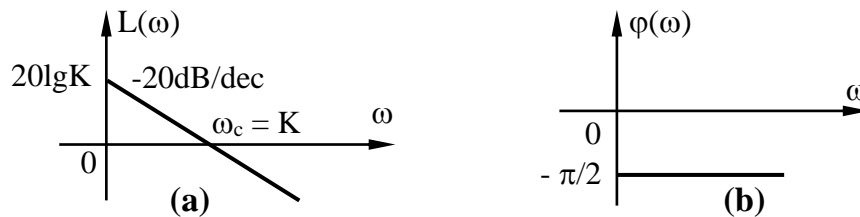


Figure 2.31. Logarithmic characteristics of frequency amplitude and frequency phase characteristics

From the time characteristics of the integration stage, we see that the integration stage has memory properties. That is, the integration stage will remain in the same state at the time the input stops.

The transition function of the integral phase increases to infinity, so the integral phase tends to cause instability.

The phase-frequency characteristics of the integration stage show that the output signal is always slower in phase than the input signal by an angle of $\pi/2$.

2.2.4. Differential stitching

- Differential equations

The differential step is a typical dynamic step where the output quantity is proportional to the derivative of the input quantity.

The differential step is considered in two cases:

Ideal differential:

$$y(t) = K \cdot \frac{du(t)}{dt} \quad (2.78)$$

First order differential stage:

$$y(t) = K \cdot \left[T \frac{du(t)}{dt} + u(t) \right] \quad (2.79)$$

In which: T is the differential time constant, K is the transmission coefficient.

Examples of elements equivalent to the differential stage: Transformer, RL four-pole network, CR four-pole network.

- Transfer function

Ideal differential:

$$W(p) = \frac{Y(p)}{U(p)} = K \cdot p \quad (2.80)$$

First order differential stage:

$$W(p) = K(Tp + 1) \quad (2.81)$$

- Time characteristics

+ Transient function

Ideal differential:

$$h(t) = K \frac{dl(t)}{dt} = K \cdot \delta(t) \quad (2.82)$$

First order differential stage:

$$h(t) = K \cdot l(t) + KT \frac{dh(t)}{dt} = K \cdot l(t) + KT \cdot \delta(t) \quad (2.83)$$

+ Weight function

Ideal differential:

$$g(t) = \frac{dh(t)}{dt} = K \frac{d\delta(t)}{dt} \quad (2.84)$$

First order differential stage:

$$g(t) = K \left[T \frac{d\delta(t)}{dt} + \delta(t) \right] \quad (2.85)$$

The timing characteristics of an ideal differential step are described in Figure 2.32.

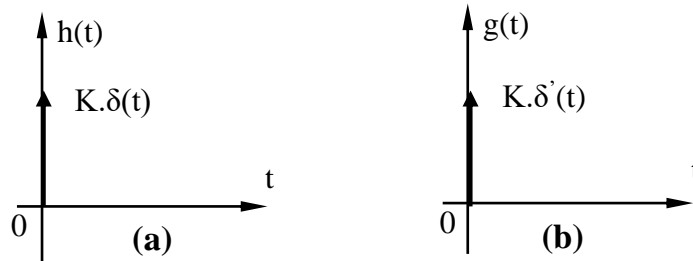


Figure 2.32. Time characteristics of the ideal differential step

- Frequency characteristics

+ Frequency phase amplitude characteristics

Ideal differential step:

$$W(j\omega) = W(p)|_{p=j\omega} = jK\omega \quad (2.86)$$

First order differential stage:

$$W(j\omega) = K + j\omega KT \quad (2.87)$$

+ Frequency amplitude characteristics

Ideal differential:

$$A(\omega) = K.\omega \quad (2.88)$$

First order differential stage:

$$A(\omega) = K\sqrt{1 + \omega^2 T^2} \quad (2.89)$$

+ Frequency amplitude logarithmic characteristics

Ideal differential:

$$L(\omega) = 20\lg K\omega \quad (2.90)$$

First order differential stage:

$$L(\omega) = 20\lg K + 20\lg \sqrt{1 + \omega^2 T^2} \quad (2.91)$$

The $L(\omega)$ characteristic is built similarly to the inertial link:

$$L(\omega) = \begin{cases} 20\lg K & 0 < \omega \leq \frac{1}{T} \\ 20\lg K + 20\lg \omega T & \omega > \frac{1}{T} \end{cases} \quad (2.92)$$

+ Frequency phase characteristics

Ideal differential:

$$\varphi(\omega) = \arctg \frac{K}{0} = \frac{\pi}{2} \quad (2.93)$$

First order differential stage:

$$\varphi(\omega) = \arctg\omega T \quad (2.94)$$

The frequency characteristics of the ideal differential and first-order differential are described in Figure 2.33 and Figure 2.34, respectively.

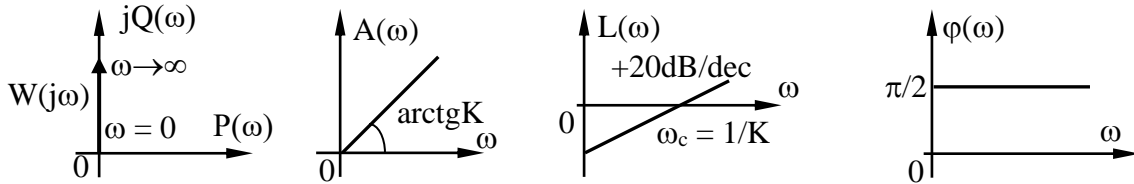


Figure 2.33. Frequency characteristics of the ideal differential stage

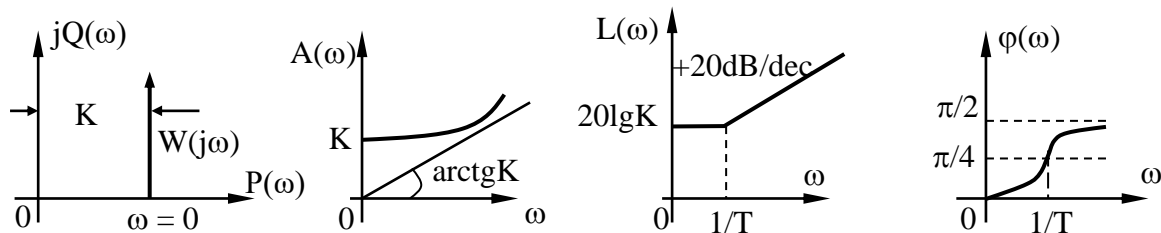


Figure 2.34. Frequency characteristics of the first-order differential stage

The temporal characteristics of the differential stage show that the differential stage tends to cause instability, because its transient function and weight function increase to infinity.

The differential step has the output signal always ahead of the input signal by an angle equal to $\pi/2$ (ideal differential) or an angle varying from $0 \div \pi/2$ according to the change in ω (first order differential). This is an outstanding feature of the differential stage that makes the system act quickly.

2.3. CHAPTER 2 EXERCISES

2.3.1. Sample solution exercises

Exercise 1. Find the transition function of the system with the following transfer function:

$$W(p) = \frac{H(p)}{U(p)} = \frac{K}{Tp + 1 + K}$$

The answer:

When $u(t) = 1(t)$ then $U(p) = 1/p$. Then:

$$H(p) = \frac{K}{Tp + 1 + K} \cdot \frac{1}{p}$$

The inverse Laplace transform is found:

$$h(t) = L^{-1}\{H(p)\} = L^{-1}\left\{\frac{K}{Tp + 1 + K} \cdot \frac{1}{p}\right\}$$

Or:
$$h(t) = L^{-1} \left\{ \frac{K}{1+K} \cdot \frac{1}{p} - \frac{K}{1+K} \cdot \frac{1}{p + (1+K)/T} \right\}$$

Looking up the Laplace transform table, we get:

$$h(t) = \frac{K}{1+K} (1 - e^{-t/T_1}) \text{ khi } t \geq 0$$

In there: $T_1 = \frac{T}{1+K}$

The transient characteristics are described in Figure 2.35.

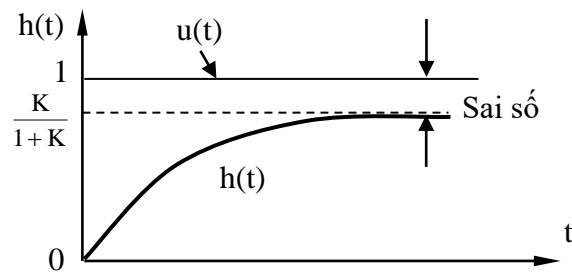


Figure 2.35. Transient characteristic h(t)

Exercise 2. Determine the transfer function of an open system with a Bode plot as depicted in Figure 2.36.

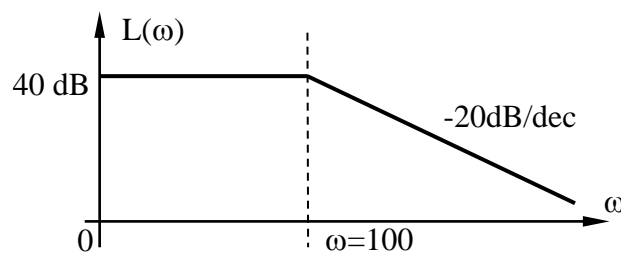


Figure 2.36. Amplitude Bode plot of open system

The answer:

The transfer function has the form:

$$W(p) = \frac{K}{Tp + 1}$$

In there:

$$20 \log K = 40 \Rightarrow K = 100;$$

$$T = \frac{1}{\omega} = \frac{1}{100} = 0,01.$$

So the transfer function of the open system is:

$$W(p) = \frac{100}{0,01p + 1}.$$

2.3.2. Self-explanatory exercises

1. Find the impulse response of the system with the following transfer function

$$W(p) = \frac{1}{(p+1)^2(p+2)}$$

2. Determine the transition function for the system with the following transfer function

$$W(p) = \frac{5(1-0,4p)}{(p+1)(0,2p+1)}$$

3. Determine the output response for the system with the following transfer function, with $u(t) = e^{-t}$

$$W(p) = \frac{p^2 + 9p + 18}{p^2 + 6p + 8}$$

4. Give the command transmission function and the missile command transmission line:

$$W_\lambda(p) = \frac{K_\lambda}{1 + pT_\lambda}$$

With $K_\lambda = 15$ and $T_\lambda = 0.04$, please tell us what step the above transfer function is equivalent to? Find the transition function and draw the Nyquist diagram of the system?

5. Draw the amplitude Bode and phase Bode plots of the system with the following transfer function

$$W(p) = \frac{100(0,1p+1)}{p(0,01p+1)}$$

6. Give the missile's target angular coordinate tracking system as described in Figure 2.37

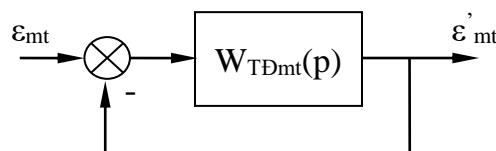


Figure 2.37. Missile target angular coordinate tracking system

In there $W_{TĐmt}(p) = K_1 \frac{(1 + pT_1)}{p(1 + pT_2)}$. Identify the typical kinetic stages that

create $W_{TĐmt}(p)$.

Let $K_1 = 5$; $T_1 = 0.1$; $T_2 = 0.08$; Find the weight function and draw the Nyquist diagram of the closed system in Figure 2.37.

7. Give three transient characteristic curves of the missile as depicted in figure 2.38, in which: Line (1) is a missile with static stability, lacking ASS; track (2) is a missile with no static stabilization and lacks ASS; path (3) is a missile with ASS (with ASS being the on-board stabilization system).

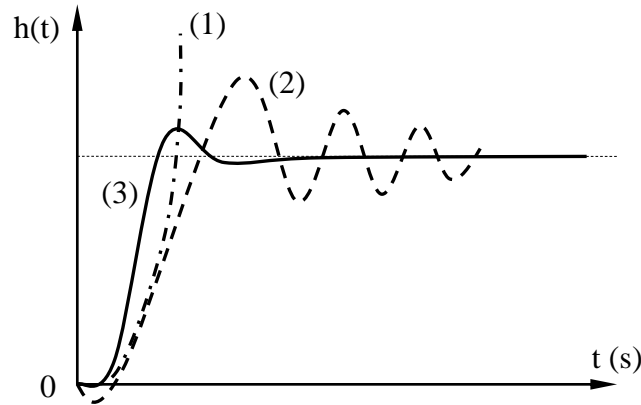


Figure 2.38. Transit characteristics of rockets

From the point of view of automatic control, let's compare the three transient characteristic curves.

8. Draw the Nyquist plot of the system with the following transfer function

$$W(p) = \frac{2p^2 + 5p + 1}{p^2 + 2p + 3}$$

9. Show the block diagram of the satellite navigation system as depicted in Figure 2.39. In essence, this system will control the satellite's telescope to always follow a certain object in space regardless of interfering factors. Suppose the natural oscillation frequency of the system is $\omega_n = K_t K_a K_b J^{-1}$. Find the transfer function and transition function of the system.

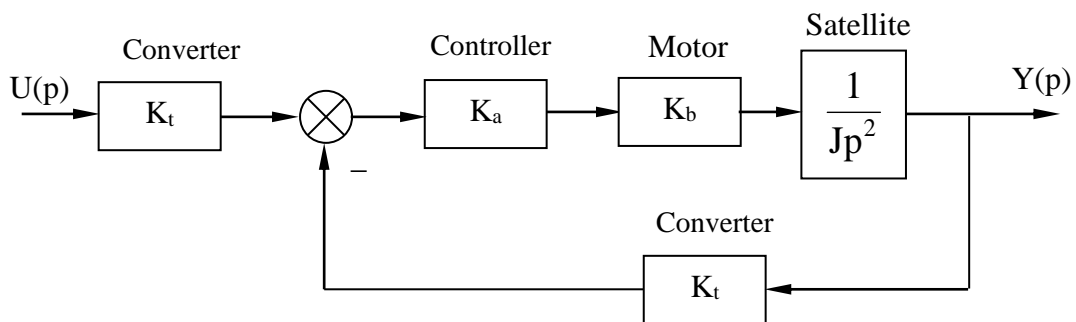


Figure 2.39. Satellite navigation system

Answer :

$$W(p) = \frac{K_t K_a K_b}{Jp^2 + K_t K_a K_b} = \frac{\omega_n^2}{p^2 + \omega_n^2}$$

10. Use the inverse Laplace transform to find the transient response of the system when the input signal is a function of $1(t)$.

$$W(p) = \frac{K(p + 2)}{(p + 1)(p + 3)}$$

Conclusion of chapter 2

This chapter has presented the concept of the dynamic characteristics of a linear automatic control system, including time characteristics and frequency characteristics. This chapter also presents how to build the dynamic characteristics of the basic stages and the system. Students must master the dynamic characteristics of the basic stages and how to build the dynamic characteristics of the system to be able to well solve the problem of analyzing, surveying, and evaluating the automatic control system that will be presented in the following chapters.

Chapter 2 review questions

Question 1: Why is it called a typical signal? What are those signals?

Question 2: Analyze the response and dynamic characteristics of the system.

Question 3: Differential equations and transfer functions of typical kinematic stages.

Question 4: Analyze the time characteristics of typical dynamic stages.

Question 5: Analyze the frequency characteristics of typical dynamic stages.

Chapter 3

SURVEY OF THE STABILITY OF THE AUTOMATIC CONTROL SYSTEM

The first requirement for an automatic control system is that the system must maintain a stable state when affected by input signals and influenced by noise on the system. For linear systems, the characteristics of the transient process do not depend on the value of the stimulus. The stability of a linear automatic control system does not depend on the type or value of the input signal, and in a linear system there exists only one equilibrium state. This chapter deals with methods to consider the stability of linear automatic control systems according to standards.

3.1. GENERAL OVERVIEW OF THE STABILITY OF AUTOMATIC CONTROL SYSTEMS

3.1.1. Stability of the automatic control system

a) Definition

Stability investigation is one of the important tasks when analyzing a system. That task always comes before quality assessment. Because an unstable system is not capable of working, there is no longer a duty to evaluate quality.

Example 3.1.1. Consider the state of the sphere located at three different positions relative to the terrain as depicted in figure 3.1.

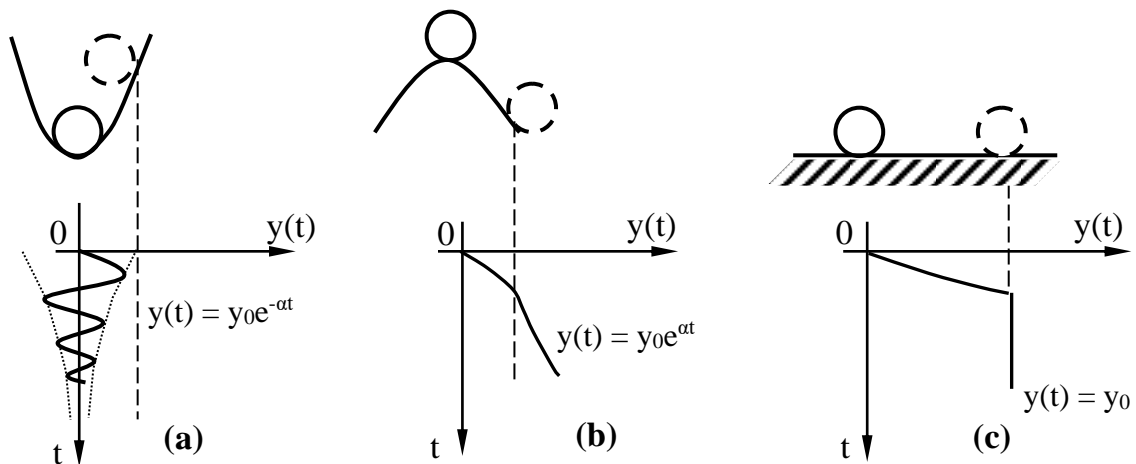


Figure 3.1. The state of the sphere located in three different positions

When an external force is applied to the sphere, it will change position.

For the case of Figure 3.1a, we say the system is stable because: after the effect of the external force ends, after a certain period of oscillation around the equilibrium position, the ball will return to its original position with the smallest potential energy.

In the case of Figure 3.1b, we say the system is unstable because the ball dropped from the top is not able to return to its original position when the effect of the external force ends.

In the case of Figure 3.1c, we say the system is at the stable boundary because, after the effect of the external force ends, the relative position between the ball and the terrain is the same as the original.

Thus, we can define stability as:

The stability of a system is the ability to automatically return to a pre-established state after the impact that disrupts that established state is lost.

b) Mathematical basis for system stability

Investigating the stability of a linear automatic control system can be done through analyzing mathematical expressions describing the dynamic process of the system.

The dynamic properties of an automatic control system are often represented by a general differential equation of the form:

$$\sum_{i=0}^n a_i \frac{d^{n-i}y(t)}{dt^{n-i}} = \sum_{j=0}^m b_j \frac{d^{m-j}u(t)}{dt^{m-j}} \quad (3.1)$$

In which:

$u(t)$, $y(t)$ are the input quantity and output quantity,

a_i , b_j are the coefficients of the differential equation ($i = 0, \dots, n$; $j = 0, \dots, m$).

Equation (3.1) describes a system consisting of two processes, a steady state and a transition process, characterized by solutions:

$$y(t) = y_0(t) + y_{qd}(t) \quad (3.2)$$

In there:

$y_0(t)$ is a particular solution of (3.1) when there is a right-hand side, characterizing the establishment process.

$y_{qd}(t)$ is the general solution of (3.1) when the right side is zero, characterizing the transition process.

process is always stable, so the stability of the system depends only on the transient solution. Thus, the remaining problem only needs to consider the transient process $y_{qd}(t)$. If the system is to be stable, the transient solution must disappear over time.

So the condition for the system to be stable is:

$$\lim_{t \rightarrow \infty} y_{qd}(t) = 0 \quad (3.3)$$

To have a basis for considering stability, we have the following definitions:

A stable system is one that has a gradual damping transition over time.

An unstable system is one whose transitions gradually increase over time.

A system at the stable boundary is a system with a constant transition or undamped oscillation.

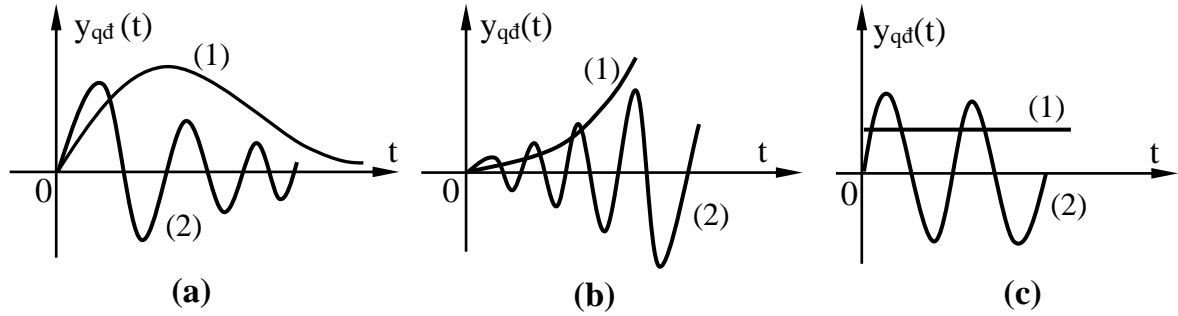


Figure 3.2. Transient process of the control system

Figure 3.2 illustrates the transition process with the cases of a stable system (Figure 3.2a), an unstable system (Figure 3.2b), and a system at the stable border (Figure 3.2c), in which there are two characteristic lines: line (1) is a non-oscillating system, and line (2) is an oscillating system.

In essence, determining the stability of the system means finding the solution to the differential equation describing the state of the system. The solution of the differential equation includes two components, in which the transition solution has the form:

$$y_{qd}(t) = \sum_{i=1}^n C_i e^{p_i \cdot t} \quad (3.4)$$

In which: C_i is the integration constant;

p_i is the solution of the characteristic equation:

$$A(p) = a_0 \cdot p^n + a_1 \cdot p^{n-1} + \dots + a_n = 0 \quad (3.5)$$

Now according to (3.3) we have:

$$\lim_{t \rightarrow \infty} y_{qd}(t) = \lim_{t \rightarrow \infty} \sum_{i=1}^n C_i e^{p_i \cdot t} \quad (3.6)$$

We consider p_i in expression (3.6) to be the solution of the characteristic equation (3.5), it can be a real solution or a conjugate complex solution.

When p_i are real roots $p_i = \alpha_i$:

$$\lim_{t \rightarrow \infty} y_{qd}(t) = \lim_{t \rightarrow \infty} \sum_i C_i \cdot e^{\alpha_i t} = \begin{cases} 0 & \text{if } \alpha_i < 0 \text{ The system is stable} \\ \sum_i C_i & \text{if } \alpha_i = 0 \text{ The system is at the stable boundary} \\ \infty & \text{if } \alpha_i > 0 \text{ The system is unstable} \end{cases} \quad (3.7)$$

The illustrative graph of expression (3.7) is depicted in figure 3.3.

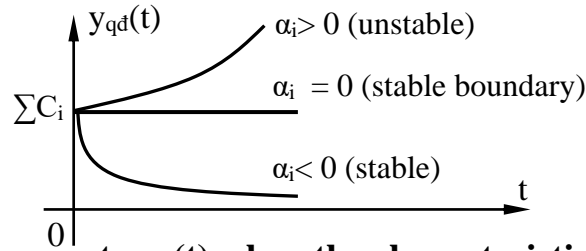


Figure 3.3. Represent $y_{qd}(t)$ when the characteristic equation has real solutions

When p_i are pairs of conjugate complex solutions $p_{i,i+1} = \alpha_i \pm j\omega_i$:

Since p_i is a pair of conjugate complex roots, the transition solution is equal to:

$$y_{qd(i,i+1)}(t) = y_{qd,i}(t) + y_{qd,i+1}(t) = 2C_i e^{\alpha_i t} \cos(\omega_i t) \quad (3.8)$$

Then, we have:

$$\lim_{t \rightarrow \infty} y_{qd}(t) = \lim_{t \rightarrow \infty} \sum_i 2C_i e^{\alpha_i t} \cos(\omega_i t) = \begin{cases} 0 & \text{if } \alpha_i < 0 \text{ The system is stable} \\ \sum_i 2C_i \cos(\omega_i t) & \text{if } \alpha_i = 0 \text{ The system is at the stable boundary} \\ \infty & \text{if } \alpha_i > 0 \text{ The system is unstable} \end{cases} \quad (3.9)$$

The graphical representation of expression (3.8) is depicted in figure 3.4.

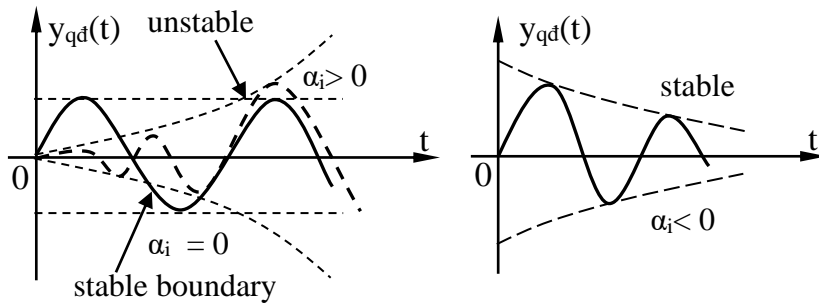


Figure 3.4. Represent $y_{qd}(t)$ when the characteristic equation has pairs of complex solutions

Example 3.1.2. Given a control object that is a rocket with a transfer function equal to:

$$W_{TL}(p) = \frac{-K}{T^2 p^2 + 2\xi T p + 1}$$

In which:

K is the transmission coefficient, T is the time constant ($T > 0$), ξ is the attenuation coefficient ($0 < \xi < 1$). Let's examine the stability of the rocket.

The answer: We have the characteristic equation.

$$A(p) = T^2 p^2 + 2\xi Tp + 1 = 0$$

Solve the above characteristic equation to find the pair of complex solutions:

$$p_{1,2} = -\frac{\xi}{T} \pm j \frac{\sqrt{1-\xi^2}}{T}$$

Then the transient solution is calculated according to formula (3.8) by:

$$y_{q\ddot{d}}(t) = 2.e^{\left(-\frac{\xi}{T}t\right)}. \cos\left(\frac{\sqrt{1-\xi^2}}{T}t\right)$$

Using expression (3.9) we have:

$$\lim_{t \rightarrow \infty} y_{q\ddot{d}}(t) = \lim_{t \rightarrow \infty} 2.e^{\left(-\frac{\xi}{T}t\right)}. \cos\left(\frac{\sqrt{1-\xi^2}}{T}t\right) = 0$$

So by definition, the above missile system is stable.

Here we draw the following conclusions:

The system is stable if all the roots of the characteristic equation have a negative real part (all the roots lie on the left side of the complex plane), i.e.:

$$\text{Re}\{p_i\} < 0 \text{ with } \forall i = \overline{1, n}$$

The system is unstable if there is only one solution of the characteristic equation with a positive real part (at least one solution lies on the right side of the complex plane, the remaining solutions lie on the left side of the complex plane), that is:

$$\exists i \in (\overline{1, n}) \text{ but } \text{Re}\{p_i\} > 0$$

The system is at the border of stability if there is only one solution with zero real part, the remaining solutions have negative real part (at least one solution lies on the imaginary axis and the remaining solutions are on the left of the complex plane), it mean:

$$\exists j \in (\overline{1, n}) \text{ but } \text{Re}\{p_j\} = 0$$

$$\text{Also } \forall i \in (\overline{1, n}), i \neq j \text{ - } \text{Re}\{p_i\} < 0$$

On the solution distribution plane of the characteristic equation described in Figure 3.5, real solutions lie on the real axis Re , purely imaginary solutions $p = j \omega$ lie on the imaginary axis Im , solutions with negative real parts are all located at left half, and the solutions with positive real part are all located in the right half of the complex plane.

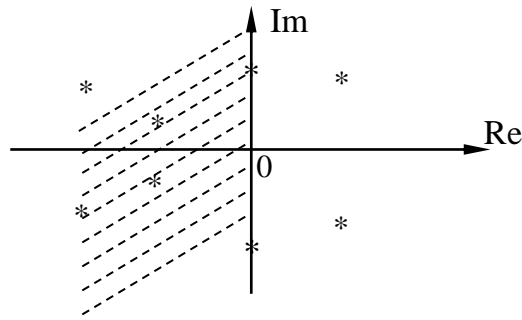


Figure 3.5. Solution distribution on the complex plane

Based on that distribution, we can state the stability conditions of the system as follows:

The necessary and sufficient condition for a linear automatic control system (closed or open) to be stable is that all solutions of the characteristic equation lie on the left half of the complex plane. If there is even one solution that lies to the right of the complex plane, the system is unstable. If there is even one solution located on the imaginary axis and the other solutions lie on the left half of the complex plane, then the system is at the boundary of stability.

Thus, the virtual axis Im divides the solution plane into two regions: the left is the stable region, the right is the unstable region, and the virtual axis is the stable boundary.

The disadvantage of the method of solving differential equations to evaluate the stability of the system is that it requires a lot of mathematical transformation tricks, creating a large amount of work, especially when solving hierarchical high differential equations.

To overcome the above shortcomings, in practice, people use indirect stability assessment methods called stability standards. Often, use the following standards: Algebraic standards and frequency standards.

The advantage of the above criteria is that there is no need to solve the differential equation, but it is still possible to determine both the cause of instability and the ability to ensure its stability.

The above criteria are all based on how the solution of the characteristic equation $A(p) = 0$ is distributed on the complex plane to conclude the stability of the system.

3.1.2. Necessary conditions for the automatic control system to be stable

a) State the necessary conditions

A system can always be described by a characteristic equation of the form:

$$a_0 \cdot p^n + a_1 \cdot p^{n-1} + \dots + a_n = 0 \quad (3.10)$$

In which: n is the highest order of the characteristic equation (also the

highest order of the tissue differential equationsystem description); a_i are the coefficients of the characteristic equation ($i = \overline{1, n}$).

Stated :

The necessary condition for the automatic control system to be stable is that all coefficients of the characteristic equation are positive ($a_i > 0$).

b) Prove

According to mathematical basis, if the system is stable, the solutions of the characteristic equation will be:

$$p_1 = -\alpha_1; p_{2,3} = -\alpha_2 \pm j\beta_2; \dots; p_n = -\alpha_n$$

In there: $\alpha_i > 0$ ($i = \overline{1, n}$)

Then are the coefficients of the characteristic equation positive or not?

Indeed, if the characteristic equation (3.10) has n solutions, it can be expressed as:

$$a_0(p - p_1)(p - p_2)\dots(p - p_n) = 0 \quad (3.11)$$

Substituting the above solution values we get:

$$a_0(p + \alpha_1)[(p + \alpha_2 - j\beta_2)(p + \alpha_2 + j\beta_2)]\dots(p + \alpha_n) = 0 \quad (3.12)$$

Continuing to expand and arrange in the form (3.10), we have:

$$a'_0 p^n + a'_1 p^{n-1} + \dots + a'_n = 0 \quad (3.13)$$

The coefficients $a'_0; a'_1; \dots; a'_n$ in expression (3.13) are always positive because the terms in (3.12) are positive and the calculation involves only addition and square.

Therefore, when the system is stable, the coefficients of the characteristic equation must be positive.

Example 3.1.3 . Consider the stability of systems with the following characteristic equations:

a) $p^3 + 6p^2 + 3p + 4 = 0$, the system can be stable.

b) $p^3 + 5p^2 - 3p + 6 = 0$, the system is unstable, because $a_2 = -3$.

c) $3p^3 + 4p + 2 = 0$, the system is unstable, because $a_1 = 0$.

Conclusion: The coefficients (a_i) of the characteristic equation are created by the values of the system parameters, so the stability of the system is determined through the parameters and does not depend on the state. of the system.

3.2. ALGEBRAIC STABILITY CRITERIA

3.2.1. Routh standard

a) State the Routh criterion

Suppose a system is described by the following n^{th} -order characteristic equation:

$$a_0 \cdot p^n + a_1 \cdot p^{n-1} + \dots + a_{n-1} \cdot p + a_n = 0 \quad (3.14)$$

Then the Routh table has the form:

a_0	a_2	a_4	a_6	a_8	...
a_1	a_3	a_5	a_7	a_9	...
b_0	b_2	b_4	b_6	...	
b_1	b_3	b_5	b_7	...	
...		
z_0					
z_1					

Stated: The necessary and sufficient condition for a linear automatic control system to be stable is that all terms in the first column of the Routh table are positive.

b) How to create a Routh table

The first two rows of the Routh table contain the coefficients of the characteristic equation arranged as follows: the first row records terms with even indices starting from a_0 , the second row records terms with odd indices starting from a_1 . Detail:

a_0	a_2	a_4	a_6	a_8
a_1	a_3	a_5	a_7	a_9

From the third row of the Routh table onwards, each term is a fraction:

Numerator: L is a quadratic determinant with a negative sign, with the first column of the determinant also being the first column of the two rows next to the row with the term being calculated, and its second column is the column next to it. To the right, the term being calculated is also from the two rows above.

Denominator: All terms of a row have the same denominator, which is the first term of the row immediately above the row being calculated.

The Routh table has $n+1$ (n is the highest order of the characteristic equation). The number of elements in each row of the table will gradually decrease until the last two rows have one element left. Detail:

$$b_0 = \frac{-\begin{vmatrix} a_0 & a_2 \\ a_1 & a_3 \end{vmatrix}}{a_1}, b_2 = \frac{-\begin{vmatrix} a_0 & a_4 \\ a_1 & a_5 \end{vmatrix}}{a_1}; \dots b_1 = \frac{-\begin{vmatrix} a_1 & a_3 \\ b_0 & b_2 \end{vmatrix}}{b_0}, b_3 = \frac{-\begin{vmatrix} a_1 & a_5 \\ b_0 & b_4 \end{vmatrix}}{b_0}; \dots$$

$$c_0 = \frac{-\begin{vmatrix} b_0 & b_2 \\ b_1 & b_3 \end{vmatrix}}{b_1}, c_2 = \frac{-\begin{vmatrix} b_0 & b_4 \\ b_1 & b_5 \end{vmatrix}}{b_1}; \dots c_1 = \frac{-\begin{vmatrix} b_1 & b_3 \\ c_0 & c_2 \end{vmatrix}}{c_0}, c_3 = \frac{-\begin{vmatrix} b_1 & b_5 \\ c_0 & c_4 \end{vmatrix}}{c_0}; \dots$$

c) Properties of the Routh table

It is possible to multiply or divide all terms in the same row of the table by a positive number, and the calculation result remains unchanged.

The number of times the signs of the terms in the first column of the table are changed is equal to the number of solutions of the characteristic equation with a positive real part (located to the right of the complex plane).

If in the first column of the table there is a coefficient equal to zero, the system is also unstable.

From the first column of the table, we can calculate the critical value of a certain variable parameter in the system (for example, the amplification coefficient).

The Routh standard can be applied to consider stability for both closed and open systems with characteristic equations of any order.

If the system contains a delay with a transfer function of the form e^{-pT} , then we use the approximate Taylor expansion formula as follows:

$$e^{-pT} = 1 + \frac{-pT}{1!} + \frac{(-pT)^2}{2!} + \frac{(-pT)^3}{3!} + \dots$$

Then we take the first two terms:

$$e^{-pT} \approx 1 - pT \quad (3.15)$$

Example 3.2.1. Consider the stability of the system with the following characteristic equation:

$$p^4 + 2p^3 + 8p^2 + 4p + 3 = 0$$

The answer:

Creating a Routh table based on the coefficients of the characteristic equation, we have:

1	8	3
2	4	0
b_0	b_2	
b_1		
c_0		

Calculate the terms from the third row onwards:

$$b_0 = \frac{-\begin{vmatrix} 1 & 8 \\ 2 & 4 \end{vmatrix}}{2} = 6, \quad b_2 = \frac{-\begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix}}{2} = 3, \quad b_1 = \frac{-\begin{vmatrix} 2 & 4 \\ 6 & 3 \end{vmatrix}}{6} = 3, \quad c_0 = \frac{-\begin{vmatrix} 6 & 3 \\ 3 & 0 \end{vmatrix}}{3} = 3.$$

Then, we have a complete Routh table as follows:

1	8	3
2	4	0
6	3	
3		
3		

We see that the terms in the first column of the Routh table are all positive.

Therefore, the given system is stable.

d) Some special cases

- If the Routh table has the first term of a certain row equal to zero, the remaining coefficients of that row are non-zero

Now we replace that zero term by an arbitrary very small positive number ε ($\varepsilon \rightarrow 0$), then the calculation process continues. The values of the next terms depend on ε .

Example 3.2.2. Consider the stability of the system with the following characteristic equation:

$$p^4 + 2p^3 + 3p^2 + 6p + 5 = 0$$

The answer:

Creating a Routh table based on the coefficients of the characteristic equation, we have:

1	3	5
2	6	
b_0	b_2	
b_1		
c_0		

Calculate the terms from the third row onwards:

$$b_0 = \frac{-\begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix}}{2} = 0, \quad b_2 = \frac{-\begin{vmatrix} 1 & 5 \\ 2 & 0 \end{vmatrix}}{2} = 5.$$

We notice that the first term of the third row is zero, the remaining terms are non-zero. Therefore, we replace that zero term by some very small positive ε $b_0 = \varepsilon$ and continue calculating, we get:

$$b_1 = \frac{-\begin{vmatrix} 2 & 6 \\ \varepsilon & 5 \end{vmatrix}}{\varepsilon} = \frac{6\varepsilon - 10}{\varepsilon}, \quad c_0 = \frac{-\begin{vmatrix} \varepsilon & 5 \\ \frac{6\varepsilon - 10}{\varepsilon} & 0 \end{vmatrix}}{\frac{6\varepsilon - 10}{\varepsilon}} = 5.$$

Then the Routh table is found as follows:

$$\begin{array}{ccc} 1 & 3 & 5 \\ 2 & 6 & \\ \varepsilon & 5 & \\ \hline \frac{6\varepsilon - 10}{\varepsilon} & & \\ \varepsilon & & \\ 5 & & \end{array}$$

We see the term $b_1 = \frac{6\varepsilon - 10}{\varepsilon} < 0$ because ε is a very small positive number, so the first column of the Routh table has two sign changes.

Therefore, the given system is unstable and the characteristic equation has two solutions lying to the right of the complex plane.

- If the Routh table has all the terms of a certain row equal to zero

Then, establish an auxiliary polynomial $A_0(p)$ whose coefficients are the terms of the row preceding the row with all terms equal to zero.

Replace the row with all zero terms by another row whose terms are the coefficients of the polynomial $\frac{dA_0(p)}{dp}$, then the calculation continues.

Note: The solution of the auxiliary polynomial $A_0(p)$ is also the solution of the characteristic equation.

Example 3.2.3. Considering the stability of the system, the characteristic equation is:

$$p^5 + 4p^4 + 8p^3 + 8p^2 + 7p + 4 = 0$$

The answer:

Creating a Routh table based on the coefficients of the characteristic equation and calculating the terms, we have:

p^5	1	8	7
p^4	4	8	4
p^3	6	6	
p^2	4	4	
p^1	0	0	

We see that the Routh table has all terms in row p^1 equal to zero, so we can form the auxiliary polynomial as follows:

$$A_0(p) = 4p^2 + 4$$

$$\Rightarrow \frac{dA_0(p)}{dp} = 8p + 0$$

The solution of the auxiliary polynomial $A_0(p)$ is also the solution of the characteristic equation: $A_0(p) = 4p^2 + 4 = 0 \Leftrightarrow p = \pm j$

Finally we have the Routh table:

p^5	1	8	7
p^4	4	8	4
p^3	6	6	
p^2	4	4	
p^1	8		
p^0	4		

Conclude: The terms in the first column of the Routh table are positive, so the characteristic equation has no solution on the right side of the complex plane. The characteristic equation has 2 solutions located on the imaginary axis ($p = \pm j$), the remaining number of solutions lying to the left of the complex plane is $5 - 2 = 3$. Therefore, the given system is on the stable frontier.

3.2.2. Hurwitz criteria

a) State the Hurwitz criterion

Suppose a system is described by the following n^{th} -order characteristic equation:

$$a_0 \cdot p^n + a_1 \cdot p^{n-1} + \dots + a_{n-1} \cdot p + a_n = 0 \quad (3.16)$$

Then the n^{th} degree Hurwitz determinant has the form:

$$\Delta_n = \begin{vmatrix} a_1 & a_3 & a_5 & \cdots & 0 \\ a_0 & a_2 & a_4 & \cdots & 0 \\ 0 & a_1 & a_3 & \cdots & \\ \vdots & \vdots & \vdots & \vdots & \\ 0 & 0 & 0 & \cdots & a_n \end{vmatrix} \quad (3.17)$$

Stated: The necessary and sufficient conditions for a linear automatic control system to be stable are the coefficient a_0 positive and the Hurwitz determinants positive.

b) How to establish Hurwitz's determinant

The determinant Δ_n has n columns and n rows (n is the highest degree of the characteristic equation).

The main diagonal of Δ_n starts from a_1 consecutively to a_n . Terms in the same column of the Hurwitz determinant have increasing indices from bottom to top. Terms with indices higher than n and lower than zero are all zeroed.

The sub-Hurwitz determinants (from $\Delta_{n-1} \rightarrow \Delta_1$) are formed by removing columns and rows from back to front and from bottom to top.

Example 3.2.4 . Establish the Hurwitz determinants of the system with the following 4th-order characteristic equation:

$$a_0.p^4 + a_1.p^3 + a_2.p^2 + a_3.p + a_4 = 0$$

The answer: Establishing the n th-order Hurwitz determinant from the coefficients of the characteristic equation, we get:

$$\Delta_4 = \begin{vmatrix} a_1 & a_3 & 0 & 0 \\ a_0 & a_2 & a_4 & 0 \\ 0 & a_1 & a_3 & 0 \\ 0 & a_0 & a_2 & a_4 \end{vmatrix}$$

From the determinant Δ_4 , removing the columns and rows from back to front and from bottom to top, we get

$$\Delta_3 = \begin{vmatrix} a_1 & a_3 & 0 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix}, \Delta_2 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix}, \Delta_1 = a_1$$

Calculate the Hurwitz determinants $\Delta_i (i = \overline{1, n})$. We can consider the stability of the system.

c) Consequences of the Hurwitz criterion

The consequences of the Hurwitz criterion (Vunhegartxki criterion) are only considered for systems with degree $n \leq 3$.

- For the 3rd level system

The characteristic equation has the form:

$$a_0.p^3 + a_1.p^2 + a_2.p + a_3 = 0 \quad (3.18)$$

Stated:

The necessary and sufficient conditions for a stable third-order linear automatic control system are the coefficients a_i ($i = \overline{0, 3}$) positive and satisfies the inequality $a_1.a_2 > a_0.a_3$.

We can prove this consequence as follows:

The Hurwitz determinants are:

$$\Delta_1 = a_1 > 0$$

$$\Delta_2 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix} > 0 \rightarrow a_1.a_2 - a_0.a_3 > 0 \Rightarrow a_1.a_2 > a_0.a_3$$

$$\Delta_3 = \begin{vmatrix} a_1 & a_3 & 0 \\ a_0 & a_2 & 0 \\ 0 & a_1 & a_3 \end{vmatrix} = a_3.\Delta_2 > 0;$$

Since $a_3 > 0$ and $\Delta_2 > 0$, Δ_3 does not need to be calculated, the main condition is from Δ_2 .

Therefore, the stability condition for a 3rd order system is $a_1.a_2 > a_0.a_3$.

- For systems of level less than or equal to 2

The characteristic equation has the form:

$$a_0.p^2 + a_1.p + a_2 = 0 \quad (3.19)$$

Stated:

The necessary and sufficient condition for a linear automatic control system of order less than or equal to 2 to be stable is that the coefficients a_0 , a_1 , a_2 are positive.

We can use Routh or Hurwitz criteria to prove this consequence.

Note in the Hurwitz criteria: The Hurwitz stability criterion is just another representation of the Routh criterion (comparing Δ_2 in the Hurwitz criterion with the coefficient b_0 in the Routh criterion, we see that they only differ by a coefficient a_0 in the denominator).

The Hurwitz criterion is often used with systems with low-order characteristic equations ($n \leq 4$). Similar to the Routh criterion, the Hurwitz criterion can be used to find K_{th} .

The Hurwitz criterion can be applied to consider stability for both closed and open systems with characteristic equations of any order.

3.3. FREQUENCY STABILITY STANDARDS

3.3.1. Principle of application

To prove frequency stability standards, we use the following principles: rotation angle principle (Argument principle) and envelope and non-envelopment principle.

a) Principle of rotation angle

Suppose a system has the following characteristic equation:

$$A(p) = a_0 \cdot p^n + a_1 \cdot p^{n-1} + \dots + a_n = 0 \quad (3.20)$$

Because equation (3.20) has solutions p_1, p_2, \dots, p_n so the left side of (3.20) can be written as:

$$A(p) = a_0 \cdot (p - p_1) \cdot (p - p_2) \cdot \dots \cdot (p - p_n) = a_0 \cdot \prod_{i=1}^n (p - p_i)$$

Substituting $p = j\omega$ into the above expression and writing it concisely, we have:

$$A(j\omega) = a_0 \cdot \prod_{i=1}^n (j\omega - p_i) \quad (3.21)$$

With ω being the independent variable in the range $[-\infty, +\infty]$, then the factors $(j\omega - p_i)$ are complex numbers.

On the complex plane, each factor $(j\omega - p_i)$ is a vector with the origin point being p_i , the tip point being $j\omega$ ($j\omega$ runs on the imaginary axis from $[-\infty, +\infty]$).

According to (3.21) $A(j\omega)$ is the product of n complex numbers $(j\omega - p_i)$, so on the complex plane it is a vector with a rotation angle equal to the sum of the component rotation angles:

$$\Delta \arg A(j\omega) = \sum_{i=1}^n \Delta \arg (j\omega - p_i) \quad (3.22)$$

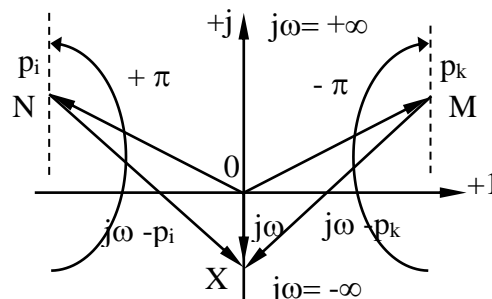


Figure 3.6. Represent the solution vector

Suppose equation (3.20) has m solutions with a positive real part (located on the right half of the imaginary axis) and $(n - m)$ solutions with a negative real part (located on the left half of the imaginary axis), as depicted in the figure 3.6.

Convention: A negative angle is an angle with clockwise rotation; a positive angle is an angle with counterclockwise rotation.

Consider the following cases of solution location on the complex plane:

If the solution p_i ($i = 1, \dots, n - m$) lies to the left of the imaginary axis:

When ω varies from $-\infty \div +\infty$ each vector $(j\omega - p_i)$ rotates counterclockwise by an angle $+\pi$:

$$\Delta \arg (j\omega - p_i) = \pi \quad (3.23)$$

$-\infty < \omega < \infty$

The total rotation angle of $(n-m)$ solutions lying to the left of the imaginary axis is:

$$\Delta \arg \prod_{i=1}^{n-m} (j\omega - p_i) = (n - m)\pi \quad (3.24)$$

$-\infty < \omega < \infty$

If the solution p_k ($k = 1, \dots, m$) lies to the right of the imaginary axis:

When ω varies from $-\infty \div +\infty$ each vector $(j\omega - p_k)$ rotates clockwise by angle $-\pi$:

$$\Delta \arg (j\omega - p_k) = \pi \quad (3.25)$$

$-\infty < \omega < \infty$

The total rotation angle of the m roots located to the right of the imaginary axis is:

$$\Delta \arg \prod_{k=1}^m (j\omega - p_k) = -m\pi \quad (3.26)$$

$-\infty < \omega < \infty$

So from (3.24) and (3.26) we have the total rotation angle of $A(j\omega)$ equal:

$$\Delta \arg A(j\omega) = \Delta \arg \prod_{i=1}^{n-m} (j\omega - p_i) + \Delta \arg \prod_{k=1}^m (j\omega - p_k) = (n - 2m)\pi \quad (3.27)$$

$-\infty < \omega < \infty$

We can rewrite (3.27) as:

$$\Delta \arg A(j\omega) = \frac{n - 2m}{2} \cdot 2\pi \quad (2.28)$$

$-\infty < \omega < \infty$

From expression (3.28), we can state the rotation angle principle as follows:

An n -order system with m roots on the right and $(n - m)$ roots on the left of the complex plane with a frequency characteristic polynomial vector $A(j\omega)$ will rotate an angle of $(n - 2m)/2$ closed loops according to counterclockwise direction when frequency ω varies from $-\infty \div +\infty$.

b) Principle of inclusion and non-inclusion

Consider point M located inside and outside the curve C as described in figure 3.7.

In the case of figure 3.7a, if a point M does not lie in a closed curve C, we say the closed curve C does not include point M, and in the case of figure 3.7b, if a point M lies in a closed curve C, we say the closed curve C covers point M.

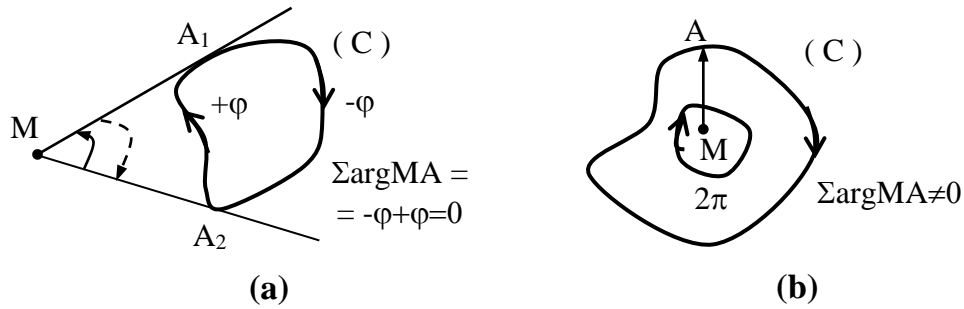


Figure 3.7. A curve that covers and does not cover a point

We can prove the following:

In figure 3.7a, from point M outside the closed curve C, we draw two tangents at A_1 and A_2 . When the endpoint A_1 of vector MA moves on the curve in the direction of the arrow to point A_2 , the vector will rotate an angle $-\varphi$, and when going from A_2 to A_1 in the other direction of the arrow, the vector MA will rotate an angle $+\varphi$. With that convention, if tip A slides across the entire circle once, the total rotation angle will be: $\sum \arg MA = -\varphi + \varphi = 0$.

Thus, if the closed curve C does not include point M, then the rotation angle of vector MA when tip A slides on the curve once must be zero.

In the case of figure 3.7b, with similar reasoning we have: If the tip A of the MA vector slides on the closed curve once, the total rotation angle of the MA vector will be 2π .

Thus, if the closed curve covers point M, the rotation angle of vector MA when tip A slides on the closed curve once must be equal to 2π . From here, it can be deduced that if the curve is closed k times covering point M, then the rotation angle of vector MA will be $2k\pi$.

3.3.2. Mikhailov standard

a) Statement of standards

The necessary and sufficient condition for a linear automatic control system to be stable is when ω varies from 0 to ∞ the Mikhailov curve $A(j\omega)$ originates from the positive real axis and covers the origin continuously in the positive direction. (counterclockwise) an angle $n\pi/2$.

In which: n is the degree of the characteristic equation $A(p) = 0$.

In Figure 3.8, the example illustrates the Mikhailop curve $A(j\omega)$ in the cases of a stable (Figure 3.8a) and unstable system (Figure 3.8b).

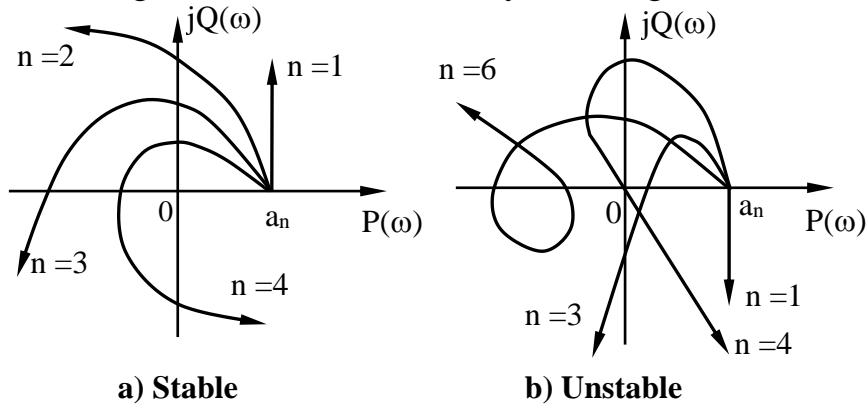


Figure 3.8. Mikhailop curve

b) Prove standards

Suppose a system whose characteristic equation is:

$$A(p) = a_0 \cdot p^n + a_1 \cdot p^{n-1} + \dots + a_n = 0 \quad (3.29)$$

Substituting $p = j\omega$ into the polynomial $A(p)$ we have:

$$A(j\omega) = a_0 (j\omega)^n + a_1 (j\omega)^{n-1} + \dots + a_{n-1} (j\omega) + a_n = P(\omega) + jQ(\omega) \quad (3.30)$$

Since $P(\omega)$ is an even function and $Q(\omega)$ is an odd function of ω , $A(j\omega)$ and $A(-j\omega)$ are two complex conjugate expressions. Inferred:

$$\Delta_{-\infty < \omega < 0} \arg A(j\omega) = \Delta_{0 \leq \omega < \infty} \arg A(j\omega) = \frac{1}{2} \Delta_{-\infty < \omega < \infty} \arg A(j\omega) \quad (3.31)$$

The system will be stable when all the solutions lie to the left of the complex plane (ie $m = 0$), from (3.28) we have:

$$\Delta_{-\infty < \omega < \infty} \arg A(j\omega) = n \cdot \pi \quad (3.32)$$

So when ω varies from $0 \div +\infty$ we find:

$$\Delta_{0 \leq \omega < \infty} \arg A(j\omega) = n \cdot \frac{\pi}{2} \quad (3.33)$$

Expression (3.33) represents the content of the Mikhailop standard.

c) Stability assessment method according to Mikhailop standard

We can evaluate the stability of a system (closed or open) according to the Mikhailop standard as follows:

Construct the graph $A(j\omega)$ by substituting $p = j\omega$ into the characteristic polynomial and then separating the real and imaginary parts:

$$A(j\omega) = P(\omega) + jQ(\omega)$$

We see that the curve $A(j\omega)$ originates from point a_n on the real axis and intersects the imaginary axis and real axis, respectively, in the positive direction (counterclockwise) with the gradual increase of ω .

When the curve $A(j\omega)$ cuts the real axis, the imaginary part is zero, $Q(\omega) = 0$. When the curve cuts the imaginary axis, the real part is zero, $P(\omega) = 0$, as illustrated in figure 3.9.

Thus, the conditions for the system to be stable are:

$$\begin{cases} a_i > 0 \\ \omega_0 < \omega_1 < \omega_2 < \dots < \omega_{n-2} < \omega_{n-1} \\ \omega_i \geq 0 \end{cases} \quad (3.34)$$

Where:

$\omega_0, \omega_2, \omega_4, \dots$ is the solution of the equation $Q(\omega) = 0$

$\omega_1, \omega_3, \omega_5, \dots$ is the solution of the equation $P(\omega) = 0$.

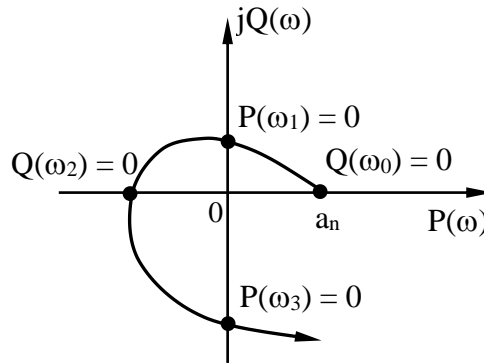


Figure 3.9. Arrange the solutions of the real and imaginary parts

Example 3.3.1. Consider the stability of the system with the following characteristic equation according to Mikhailop standard

$$A(p) = p^4 + 2p^3 + 5p^2 + 6p + 4 = 0$$

The answer:

Substituting $p = j\omega$ into the characteristic equation we have:

$$A(j\omega) = (\omega^4 - 5\omega^2 + 4) + j(6\omega - 2\omega^3) = 0$$

Solving the equation with real and imaginary parts equal to zero, taking positive frequencies we get:

$$P(\omega) = \omega^4 - 5\omega^2 + 4 = 0 \text{ has positive roots: } \omega_1 = 1 \text{ and } \omega_3 = 2$$

$$Q(\omega) = 6\omega - 2\omega^3 = 0 \text{ has positive roots: } \omega_0 = 0 \text{ and } \omega_2 = \sqrt{3}$$

We see

$$\omega_0 = 0 < \omega_1 = 1 < \omega_2 = \sqrt{3} < \omega_3 = 2 \text{ (Figure 3.10) and } a_i > 0:$$

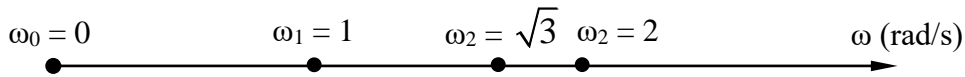


Figure 3.10. Represent the solution on the frequency axis

Satisfy the condition for the system to be stable, then the given system is stable.

3.3.3. Nyquist criterion

a) *The open system is unstable*

- Standard statements

Consider a system with unitary negative feedback, as in Figure 3.11.

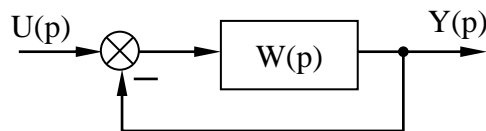


Figure 3.11. The system has unitary negative feedback

The transfer function for an open system is:

$$W(p) = \frac{B(p)}{D(p)} = \frac{\sum_{j=0}^m b_j \cdot p^{m-j}}{\sum_{i=0}^n a_i \cdot p^{n-i}} \quad (3.35)$$

The closed system transfer function will be:

$$W_k(p) = \frac{W(p)}{1 + W(p)} = \frac{B(p)}{D(p) + B(p)} = \frac{B(p)}{A(p)} \quad (3.36)$$

We always have b degree of B(p) smaller than or equal to D(p) (means $m \leq n$). Then:

$D(p) = 0$ is a typical program of an open system of order n.

$A(p) = D(p) + B(p) = 0$ is a closed system characteristic equation that also has order n.

From equations (1) and (2), we see that open and closed systems are interconnected: Both equations have n solutions, but the values of the solutions are different. Therefore, a system can be stable in an open circuit but unstable in a closed circuit, and vice versa. We can use the Nyquist criterion to consider the stability of a closed system through the phase-frequency amplitude characteristics of the open system.

In the case of an unstable open system, we can state the Nyquist criterion as follows: The necessary and sufficient condition for a closed system to be stable is that when the open system is unstable and has m roots located on the

right side of the plane complex is the frequency phase amplitude characteristic of an open system with ω variation from $0 \div +\infty$ bound point $(-1, j0)$ $m/2$ turns in the positive direction (if m is an odd number then with ω variation from $-\infty \div \infty$ characteristic frequency phase amplitude of the point-enclosed open system $(-1, j0)$ m turns in the positive direction).

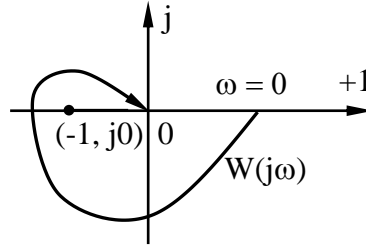


Figure 3.12. Open-system Nyquist plot covering points $(-1;j0)$

- Prove standards

Consider the complementary vector $F(j\omega)$ with the origin at $(-1, j0)$ and the tip running on the amplitude-phase-frequency characteristic chart of the open system $W(j\omega)$ as described in Figure 3.13.

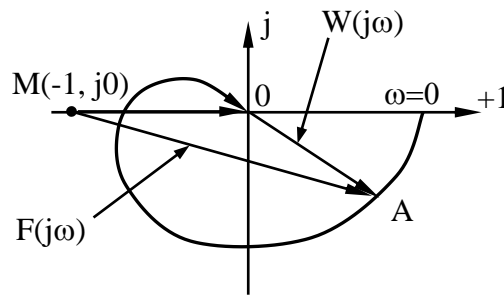


Figure 3.13. Nyquist plot with auxiliary vector $F(j \omega)$

According to the chart in Figure 3.12 we have:

$$F(j\omega) = 1 + W(j\omega) = 1 + \frac{B(j\omega)}{D(j\omega)} = \frac{D(j\omega) + B(j\omega)}{D(j\omega)} = \frac{A(j\omega)}{D(j\omega)} \quad (3.37)$$

In which: $D(j\omega)$ and $A(j\omega)$ are characteristic polynomials of open and closed systems, respectively.

Suppose the open system characteristic equation $D(p) = 0$ has m solutions located in the right half of the complex plane. According to the rotation angle principle we have:

$$\Delta \arg D(j\omega) = (n - 2m) \cdot \frac{\pi}{2} \quad (3.38)$$

$0 \leq \omega < \infty$

If the closed system wants to be stable, according to the rotation angle principle we also have:

$$\Delta \arg A(j\omega) = n \cdot \frac{\pi}{2} \quad (3.39)$$

According to (3.37) the rotation angle of vector $F(j\omega)$ is calculated by:

$$\Delta \arg F(j\omega) = \Delta \arg A(j\omega) - \Delta \arg D(j\omega) \quad (3.40)$$

Substituting expressions (3.38) and (3.39) and (3.40) we get:

$$\Delta \arg F(j\omega) = n \cdot \frac{\pi}{2} - (n - 2m) \cdot \frac{\pi}{2} = \frac{m}{2} \cdot 2\pi \quad (3.41)$$

The expression (3.41) shows that the vector $F(j\omega)$ that covers the origin $(0, j0)$ is $m/2$ round in the positive direction. Meanwhile, the graph $W(j\omega)$ is $F(j\omega)$ shifted one unit to the right. Therefore, from the chart in Figure 3.13, we see that when the open system is unstable, if the closed system wants to be stable, the amplitude phase frequency characteristics of the open system $W(j\omega)$ include the point $M(-1, j0)$ is $m/2$ loops in the positive direction when ω varies from $0 \div +\infty$.

b) The open system is stable and at the stable boundary

- Standard statements

The necessary and sufficient condition for a closed system to be stable, when the open system is stable or at the border of stability, is the phase-frequency amplitude characteristic of the open system with ω varying from $0 \div \infty$ without any point $(-1, j0)$.

- Prove standards

Proving similarly to the case of an unstable open system, we consider the support vector $F(j\omega)$ as shown in Figure 3.13.

Assuming the open system is stable, then the open system characteristic equation has n solutions on the left of the virtual axis. According to the rotation angle principle, we have:

$$\Delta \arg D(j\omega) = n \cdot \frac{\pi}{2} \quad (3.42)$$

If a closed system wants to be stable, the characteristic equation of the closed system also has n solutions on the left of the imaginary axis, so:

$$\Delta \arg A(j\omega) = n \cdot \frac{\pi}{2} \quad (3.43)$$

According to (3.40), the rotation angle of vector $F(j\omega)$ can be calculated:

$$\Delta \arg F(j\omega) = n \cdot \frac{\pi}{2} - n \cdot \frac{\pi}{2} = 0 \quad (3.44)$$

Expression (3.44) shows that the vector $F(j\omega)$ does not include the origin $(0, j0)$. Meanwhile, the graph $W(j\omega)$ is $F(j\omega)$ shifted one unit to the right.

Therefore, from the chart in Figure 3.13, we see that when the open system is stable or at the border of stability, if the closed system wants to be stable, the amplitude phase and frequency characteristics of the open system $W(j\omega)$ must not include the point $M(-1, j0)$ when ω varying from $0 \div +\infty$.

- Use conversion points.

In many cases, stable open systems or those at the stable boundary have complex phase amplitude and frequency characteristics $W(j\omega)$ and are often not closed curves (because ω only considers $0 \div +\infty$). Therefore, identifying their inclusion or non-inclusion with respect to the point $(-1, j0)$ is very difficult. To make it easier, we switch to another way of representing, which is to use transition points: positive transition points C^+ and negative transition points C^- .

The positive transition point C^+ is the transition point from negative to positive and the negative transition point C^- is the transition from positive to negative of the frequency phase amplitude characteristic $W(j\omega)$ of the open system considered on the half-line from $-\infty \div -1$ belongs to the real axis. Detail:

If $C^+ = C^-$ then the closed system is stable. Characteristic $W(j\omega)$ the open system does not include points $(-1, j0)$;

If $C^+ \neq C^-$ then the closed system is unstable. Characteristic $W(j\omega)$ open system covers the point $(-1, j0)$.

Figure 3.14 describes the case where a closed system is stable if the open system is stable or at the border of stability. We see the feature $W(j\omega)$ of an open system does not include the point $M(-1, j0)$ if the intersection of $W(j\omega)$ with the real axis in the range $(-\infty, -1)$ is absent (line 1) or is a positive conversion intersection number C^+ is equal to the number of negative transition intersections C^- (line 2 with $C^+ = C^- = 2$). Thus, the condition for $W(j\omega)$ not to include the point $M(-1, j0)$ is: $C^+ = C^-$.

Figure 3.15 describes the case where a closed system is unstable if the open system is stable or at the border of stability:

For line 1, we have $C^+ = 1, C^- = 0$; With line 2 we have $C^+ = 3, C^- = 1$. So the characteristic $W(j\omega)$ of the point-bound open system $M(-1, j0)$. Thus, the condition for $W(j\omega)$ to cover the point $M(-1, j0)$ is: $C^+ \neq C^-$.

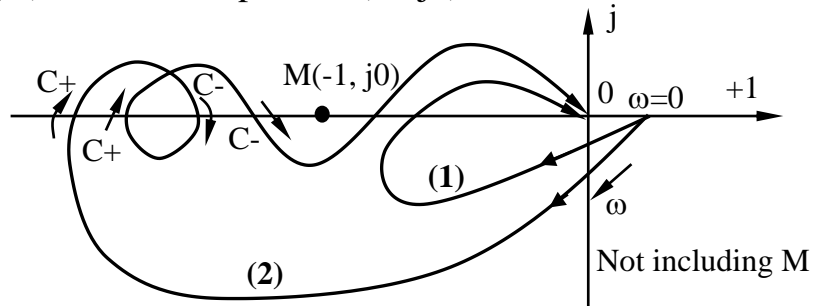


Figure 3.14. Characteristics $W(j\omega)$ of an open system without inclusion of points $M(-1, j0)$

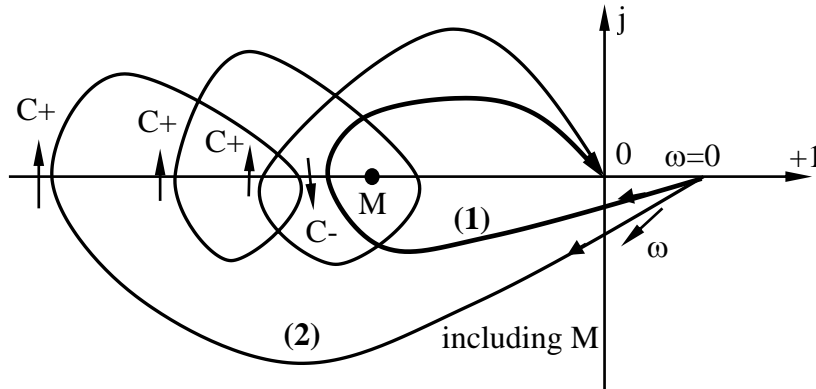


Figure 3.15. Characteristics $W(j\omega)$ of an open system inclusion of points $M(-1, j0)$

3.4. CHAPTER 3 EXERCISES

3.4.1. Sample solution exercises

Exercise 1. For a system with unit negative feedback with an open system transfer function equal to:

$$W(p) = \frac{3p+1}{3p(4p^4 + 2p^3 + 6p^2 + 2p + 1)}$$

Consider the stability of open systems according to Routh criteria, closed systems according to Mikhailop and Nyquist criteria.

The answer:

- Consider the stability of the open system

Characteristic equation of open system:

$$3p(4p^4 + 2p^3 + 6p^2 + 2p + 1) = 0$$

This equation has one solution equal to 0, and the other solutions are determined according to the equation

$$4p^4 + 2p^3 + 6p^2 + 2p + 1 = 0 \quad (*)$$

Setting up a Routh table for equation (*) we get:

p^4	4	6	1
p^3	2	2	
p^2	2	1	
p^1	1		
p^0	1		

All terms in the first column of the Routh table are positive, so the roots of this equation lie to the left of the imaginary axis.

Conclusion: The characteristic equation of an open system has one solution equal to 0 (located on the imaginary axis), the remaining solutions of this equation are all located to the left of the imaginary axis. Therefore, the open system at the boundary is stable.

- Consider the stability of the closed system
- + Consider stability according to Mikhailop standards

The transfer function of a closed system:

$$W(p) = \frac{W(p)}{1 + W(p)} = \frac{3p + 1}{3p(4p^4 + 2p^3 + 6p^2 + 2p + 1) + 3p + 1}$$

Characteristic equation of a closed system:

$$3p(4p^4 + 2p^3 + 6p^2 + 2p + 1) + 3p + 1 = 0$$

Or $12p^5 + 6p^4 + 18p^3 + 6p^2 + 6p + 1 = 0$

The coefficients $a_i > 0$, the condition needs to be guaranteed.

Substituting $p = j\omega$ into the characteristic equation of a closed system we get: $6\omega^4 - 6\omega^2 + 1 + j(12\omega^5 - 18\omega^3 + 6\omega) = 0$

Find the real solution:

$$P(\omega) = 6\omega^4 - 6\omega^2 + 1 = 0$$

Solve the equation with only non-negative solutions: $\omega_1=0,46$; $\omega_3=0,888$

Find the solution to the imaginary part:

$$Q(\omega) = 12\omega^5 - 18\omega^3 + 6\omega = 0$$

Solve the equation with only non-negative solutions: $\omega_0=0$; $\omega_2=0,5$; $\omega_4=1$

On the frequency axis, the real and imaginary part solutions are distributed alternately starting from the zero solution of the imaginary part (Figure 3.16).

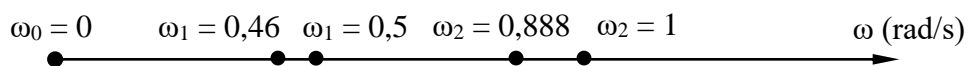


Figure 3.16. The distribution of solutions on the frequency axis

The solution distribution satisfies the stability condition, so the closed system is stable.

- + Consider stability according to the Nyquist standard

According to the results from the above section, the open system is at the stable boundary. To investigate the stability of a closed system, we must build the frequency phase amplitude characteristic $W(j\omega)$ of the open system.

Substituting $p = j\omega$ into the open system transfer function we have:

$$W(j\omega) = \frac{3j\omega + 1}{12j\omega^5 + 6\omega^4 - 18j\omega^3 - 6\omega^2 + 3j\omega} = \frac{3j\omega + 1}{6\omega^4 - 6\omega^2 + j(12\omega^5 - 18\omega^3 + 3\omega)}$$

Separating the real and imaginary parts, we get:

$$P(\omega) = \frac{36\omega^5 - 48\omega^3 + 3\omega}{\omega(6\omega^3 - 6\omega)^2 + \omega(12\omega^4 - 18\omega^2 + 3)^2}$$

$$Q(\omega) = \frac{6\omega^4 - 3}{\omega(6\omega^3 - 6\omega)^2 + \omega(12\omega^4 - 18\omega^2 + 3)^2}$$

Let $Q(\omega) = 0$ find the frequency value at which the characteristic $W(j\omega)$ of the open system intersects the real axis:

$$Q(\omega) = \frac{6\omega^4 - 3}{\omega(6\omega^3 - 6\omega)^2 + \omega(12\omega^4 - 18\omega^2 + 3)^2} = 0$$

$$\Leftrightarrow 6\omega^4 - 3 = 0 \Rightarrow \omega_a = \sqrt[4]{\frac{3}{6}} = 0,841$$

Substitute $\omega = 0,841$ into the real part $P(\omega)$:

$$P(0,841) = \frac{36(0,841)^5 - 48(0,841)^3 + 3 \cdot 0,841}{0,841 \cdot [6 \cdot (0,841)^3 - 6 \cdot 0,841]^2 + 0,841 \cdot [12 \cdot (0,841)^4 - 18 \cdot (0,841)^2 + 3]^2}$$

$$P(0,841) = -0,8383 > -1$$

So the phase-frequency amplitude characteristic of the open system (Figure 3.17) cuts the real axis in the range $-1 < -0.8383 < 0$.

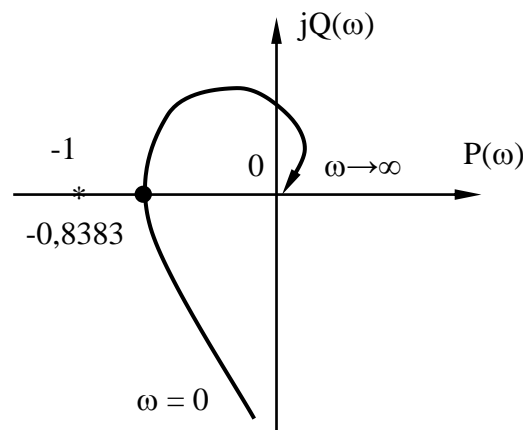


Figure 3.17. Nyquist diagram of an open system

We see that the open system frequency and phase amplitude characteristics do not include points $(-1, j0)$.

Therefore, an open system at the boundary is stable and has zero-point frequency phase amplitude characteristics $(-1, j0)$, then according to the Nyquist criterion, the closed system is stable.

Exercise 2. Consider the stability of the system with the following characteristic equation:

$$p^5 + 4p^4 + 4p^3 + 4p^2 + 5p + 3 = 0$$

The answer: Creating a Routh table from the coefficients of the characteristic equation, we have:

p^5	1	4	5
p^4	1	4	3
p^3	0	2	

Because in the Routh table the first term of a row is zero and the remaining terms are non-zero, we replace the zero term by some arbitrary very small positive number ε ($\varepsilon \rightarrow 0$) and continue calculating. maths.

Finally we have the Routh table:

p^5	1	4	5
p^4	1	4	3
p^3	ε	2	
p^2	$\frac{4\varepsilon - 2}{\varepsilon}$	3	
p^1	$\frac{3\varepsilon^2 - 8\varepsilon + 4}{4\varepsilon - 2}$		
p^0	3		

In the first column of the Routh table we see, because ε is positive very small $\rightarrow 0$, so:

The term in the 4th row in the first column is: $(4\varepsilon - 2)/\varepsilon < 0$;

The term in row 5 in the first column is:

$$\frac{3\varepsilon^2 - 8\varepsilon + 4}{4\varepsilon - 2} < 0 \text{ (due to } 3\varepsilon^2 - 8\varepsilon + 4 = \varepsilon(3\varepsilon - 8) + 4 > 0 \text{ and } (4\varepsilon - 2) < 0)$$

Thus, in the first column of the Routh table there are two terms with negative signs and two changes of sign. Therefore, the system is unstable and has two solutions with positive real parts.

3.4.2. Self-explanatory exercises

1. Determine the stability of systems with the following characteristic equations:

a) $p^4 + 10p^3 + 33p^2 + 46p + 30 = 0$

- b) $p^4 + p^3 + 3p^2 + 2p + 5 = 0$
- c) $p^3 + 2p^2 + 3p + 6 = 0$
- d) $p^5 + p^4 + 4p^3 + 4p^2 + 2p + 10 = 0$
- e) $p^4 + 5p^3 + 2p^2 + p + e^{-2p} = 0$

Answer:

- a) The system is stable
- b) The system is unstable
- c) The system at the border is stable
- d) The system is unstable
- e) The system is unstable

2. Consider the stability and determine the location of the solution on the complex plane of systems with characteristic equations as follows:

- a) $p^5 + 15p^4 + 20p^3 + 10p^2 + 5p + 100 = 0$
- b) $p^6 + 6p^5 + 13p^4 + 30p^3 + 44p^2 + 24p + 32 = 0$
- c) $p^5 + 2p^4 + 24p^3 + 48p^2 - 25p - 50 = 0$

Answer:

a) The system is unstable and the characteristic equation has 2 solutions located to the right of the imaginary axis;

b) The system is at the stable boundary and the characteristic equation has 4 solutions located on the imaginary axis;

c) The system is unstable and the characteristic equation has a solution located to the right of the imaginary axis.

3. For each characteristic equation of a control system with feedback, determine K so that the system is stable. Determine the value of K so that the system is at the boundary of stability and the oscillation frequency of the system.

- a) $p^4 + 20p^3 + 15p^2 + 2p + K = 0$
- b) $p^3 + Kp^2 + 5p + 10 = 0$

Answer:

- a) The system is stable $0 < K < 1,49$. Oscillation frequency is 0,316 rad/s
- b) Stable system $K > 2$. Oscillation frequency 2,236 rad/s.

4. The drive reader in the computer moves along the track with the diagram as shown in Figure 3.18.

Determine the conditions for K and m of the control mechanism for the system to be stable.

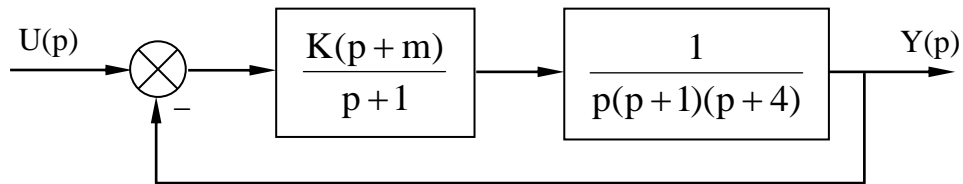


Figure 3.18. Structure diagram of computer drive reader

Answer:
$$m \leq \frac{(60 - K)(K + 6)}{36K}$$

5. Diagram of the vehicle rotation system as shown in Figure 3.19. Determine the relationship between K and a so that the system is stable.

Answer : $(K + 10)(126 - K) - 64K.a > 0$

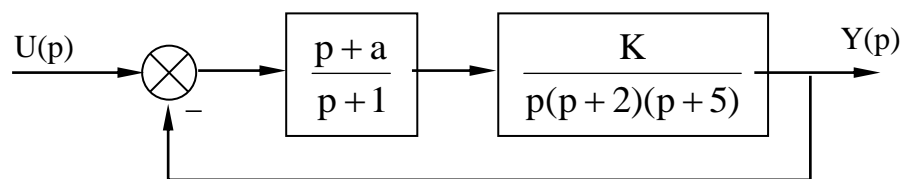


Figure 3.19. Structure diagram of the vehicle rotation system

6. The space shuttle can be used to repair satellites and the Hubble telescope. The structure diagram of that system is as shown in Figure 3.20, with $W_c(p) = K = 8$, $W(p) = \frac{1}{p(p+4)}$, $H(p) = 1$.

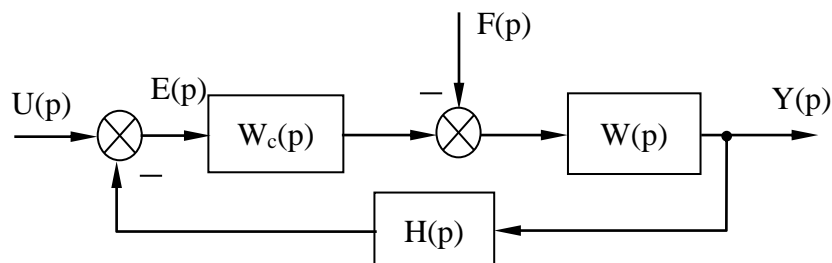


Figure 3.20. Structure diagram of the space shuttle satellite repair system

a) Determine the closed system transfer function and the noise transfer function.

b) Is the closed system stable? Why?

Answer:

a)
$$\frac{Y(p)}{U(p)} = \frac{8}{p^2 + 4p + 8}; \quad \frac{Y(p)}{F(p)} = \frac{1}{p^2 + 4p + 8}$$

b) The closed system is stable, because the characteristic equation of the closed system has all positive coefficients.

7. Give the structure diagram of the DC motor control system as shown in Figure 3.21.

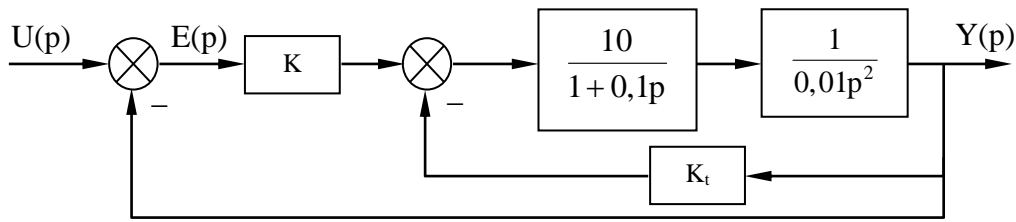


Figure 3.21. Structure diagram of DC motor control system

With $K_t = 0$; $K_t = 0,01$; $K_t = 0,1$ determines K for the system to be stable according to the Nyquist criterion.

8. Consider the stability of the submarine depth control system as shown in Figure 3.22.

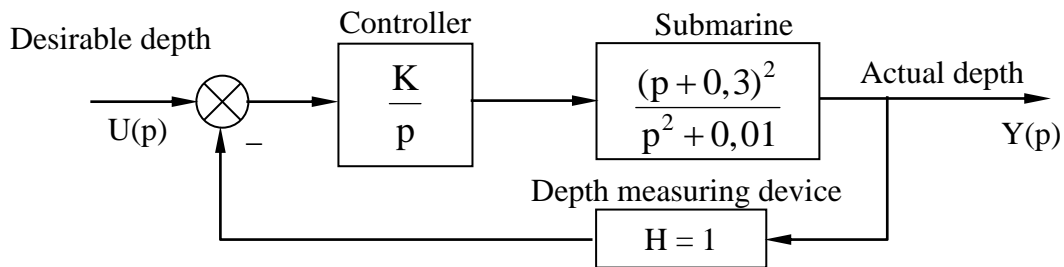


Figure 3.22. Structure diagram of the submarine depth control system

9. For the remote missile control system considered in the nodding plane, the structure diagram is as shown in Figure 3.23. Consider the stability of the system according to the given parameters.

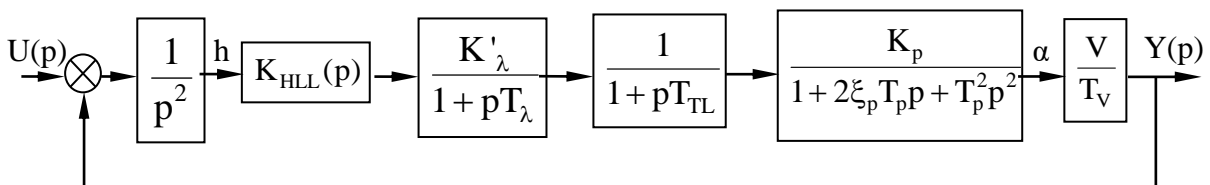


Figure 3.23. Remote missile control system in the nodding plane

10. Use the Mikhailop criterion to find the limit value of K so that the closed system in Figure 3.24 is stable.

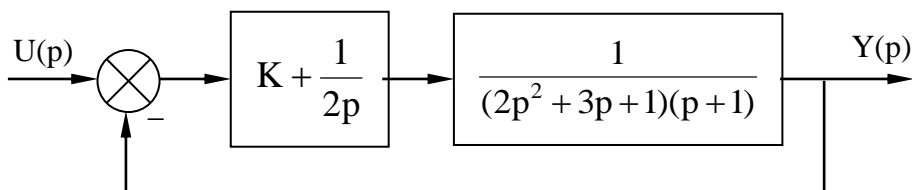


Figure 3.24. System structure diagram

Answer: $-0,33 < K < 8,33$

11. The system has a structural diagram as shown in Figure 3.25.

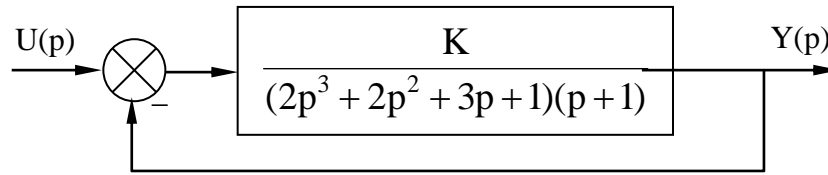


Figure 3.25. System structure diagram

Use the Nyquist criterion to determine the limit of K for the closed system to be stable.

Answer: The open system is stable; The closed system will be stable when $-1 < K < 2$.

12. A space station needs to maintain orientation between the sun and the earth to maintain power and communication. The steering system is a closed system with unit negative feedback with the actuator and control transfer function being:

$$W(p) = \frac{K(p + 20)}{p(p^2 + 24p + 144)}$$

Determine K so that the system oscillates with $\xi = 0,5$.

Answer: $K = 449,6238$.

13. The autopilot system of a supersonic aircraft helps stabilize the aerodynamics of the aircraft, thus making the flight more stable. The shortened structure diagram of this system is shown in Figure 3.26. The aircraft's dynamics are approximated by a second-order system, where K is a parameter that changes according to flight conditions. Knowing that there is no disturbance $F(s) = 0$, determine the range of values of K for the system to be stable.

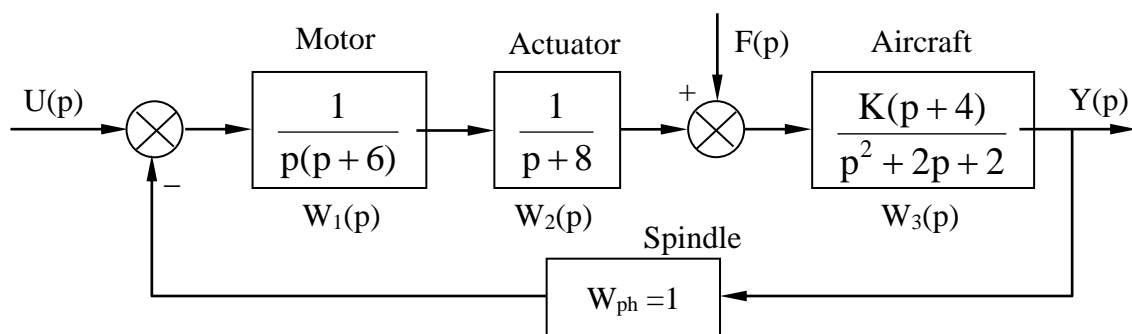


Figure 3.26. Automatic navigation system of supersonic aircraft

Answer: $0 < K < 43.3$.

14. The problem deals with the problem of controlling the flight path of a jet aircraft. A shortened diagram of a closed system is given in Figure 3.27, where the aircraft is approximated as a quaternary system. Determine the range of values of K for the closed system to be stable.

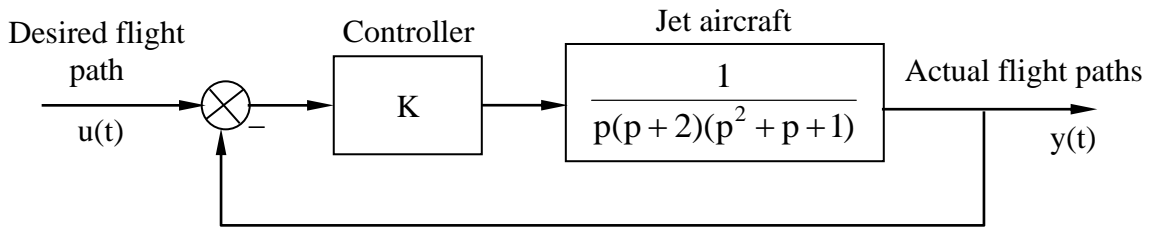


Figure 3.27. Jet aircraft flight control system

Answer: $0 < K < 9/14$.

15. Survey of a remote-controlled unmanned aerial vehicle control system for reconnaissance missions. The block diagram of the control system is depicted in Figure 3.28. Consider the stability of this control system.

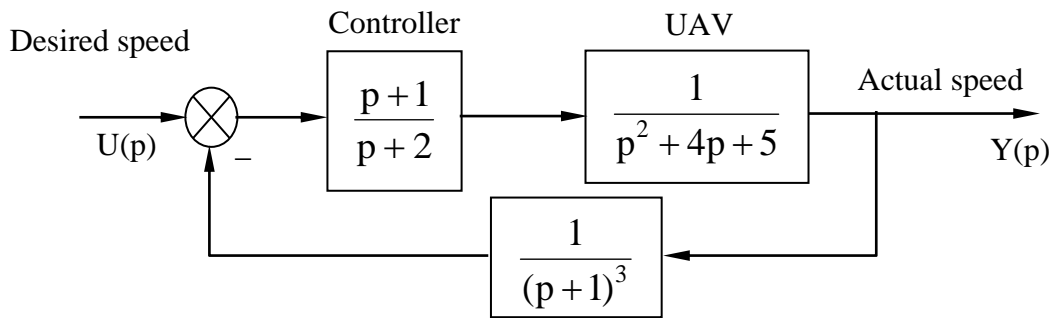


Figure 3.28. Unmanned aerial vehicle control system

16. The electromechanical system is used to stabilize and control the turret in the horizontal plane (β) as depicted in figure 3.29.

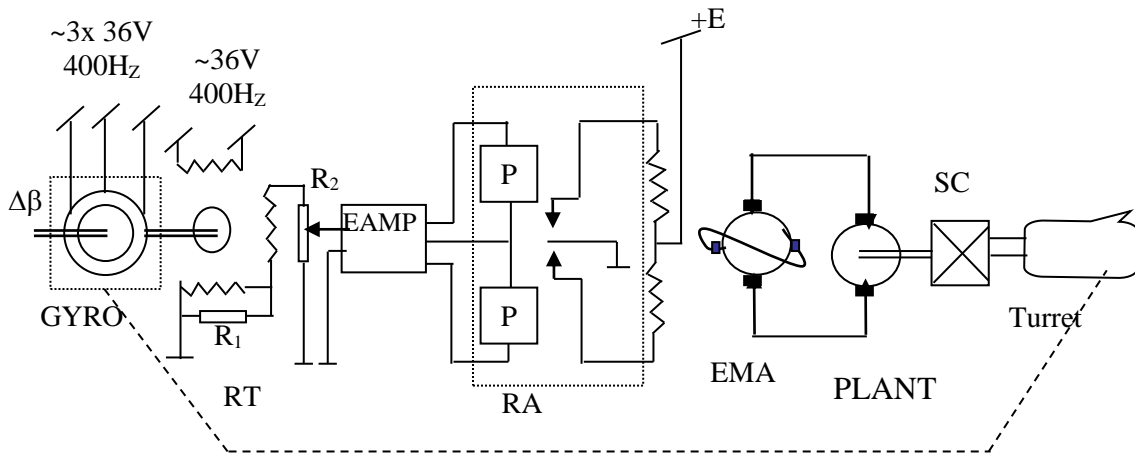


Figure 3.29. Turret stabilization and control system

In there:GYRO -Three-degree-of-freedom gyroscope used to measure the turret's deflection angles in the horizontal plane with a transfer function equal to $W_1(p) = K_1 = 1$.

RT - Rotary transformer is used to convert the error angle into an alternating voltage with a transfer function equal to $W_2(p) = K_2 = 40$.

EAMP - Phase-sensitive electronic amplifier used to pre-amplify error signals with a transfer function equal to $W_3(p) = \frac{K_3}{T_3p+1} = \frac{250}{0,007p+1}$.

RA - Relay amplifier used to amplify error signals with transfer function equal to $W_4(p) = K_4 = 1.4$.

EMA - Electrical machine amplifier used to amplify signals in terms of power with a transfer function equal to $W_5(p) = \frac{K_5}{T_5p+1} = \frac{3,2}{0,02p+1}$.

The DC electric actuator has a transfer function:

$$W_6(p) = \frac{K_6}{T_6p+1} = \frac{1,5}{0,13p+1}$$

SC - The speed changer has a transfer function equal to $W_7(p) = K_7 = 0,005$.

PLANT - The control object has a communication function equal to:

$$W_8(p) = \frac{K_8}{T_8p+1} = \frac{5}{0,25p+1}$$

Determine the stability of the above system?

Conclusion of chapter 3

To assess the stability of a system, we employ specific standards tailored to the system's characteristics. For quaternary systems, the Hurwitz standard suffices, whereas for higher-order systems, such as quinary or beyond, criteria like Routh, Mikhailov, or Nyquist must be applied. The choice of stability criteria depends on the complexity and demands of the stability problem, ensuring optimal efficiency in addressing the system's stability concerns.

Chapter 3 review questions

Question 1: The general concept of stability.

Question 2: State the algebraic stability criteria.

Question 3: How to establish the Routh table and Hurwitz determinant.

Question 4: What is the basis for considering stability according to frequency standards?

Question 5: State the frequency stability standards.

Question 6: Prove frequency stability standards.

Chapter 4

QUALITY ASSESSMENT OF THE AUTOMATIC CONTROL SYSTEM

Stability and quality constitute fundamental aspects to be examined in the study of automatic control systems. While stability is a prerequisite, the quality of a system may not necessarily meet the required standards, even if it is stable. If the system lacks stability, discussing quality becomes irrelevant. Assessing system quality involves determining whether the system operates effectively or inadequately. A quantitative evaluation is essential to thoroughly gauge the system's quality.

4.1. EVALUATING THE QUALITY OF THE SYSTEM DURING THE ESTABLISHMENT PROCESS

4.1.1. Quality criteria in the establishment process

As we know, the control process includes: Transition process (TP) and establishment process (EP). Those two processes have their own quality requirements, so to evaluate them they must also have their own criteria.

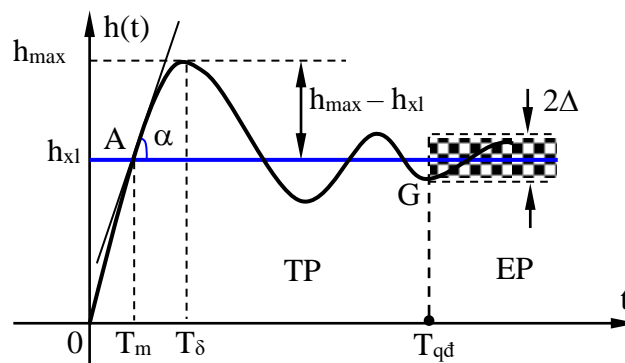


Figure 4.1. Graph of the system's transition process

The establishment process is the process when the system has an input of $u(t)$, and after the transition (state transition) ends, the system will establish a new stable state. In this new steady state, the system will have a certain error depending on the parameters and structure of the system.

The quality indicator in the establishment process is the accuracy of the system, characterized by the error of the system in the steady state:

$$e_{xl} = \lim_{t \rightarrow \infty} e(t) \quad (4.1)$$

In which:

$u(t)$ is the desired setting value, $y(t)$ is the actual setting value,

$e(t) = u(t) - y(t)$ is the established error over time.

This error exists in the stable system after the end of the transient process.

The steady state (equilibrium) of the system can be static or dynamic.

Static equilibrium is a state of equilibrium when external influences and system parameters do not change over time. Dynamic equilibrium is a state of equilibrium when external influences change according to certain rules. At this point, we say the system works in forced equilibrium mode or tracking mode. Therefore, system errors are also divided into two types: static balance errors and dynamic balance errors.

4.1.2. Calculate the steady-state error when the input signal is known

Consider a system with unit negative feedback as shown in Figure 4.2.

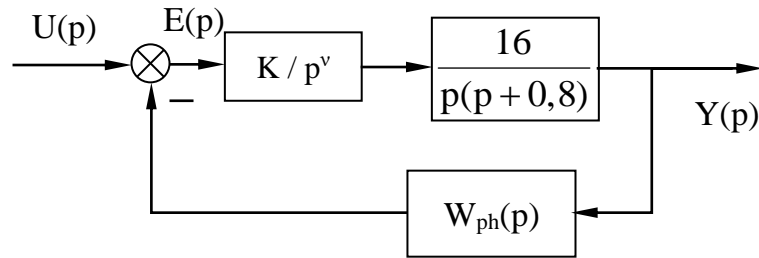


Figure 4.2. Structural diagram describing the automatic control system

From the diagram we have the open system transfer function:

$$W(p) = \frac{Y(p)}{E(p)} = \frac{K}{p^v} \cdot \frac{b_0 p^m + b_1 p^{m-1} + \dots + 1}{a_0 p^{n-v} + a_1 p^{n-v-1} + \dots + 1} \quad (4.2)$$

$$W(p) = \frac{K}{p^v} \cdot W_0(p) \quad (4.3)$$

$$W_0(p) = \frac{b_0 p^m + b_1 p^{m-1} + \dots + 1}{a_0 p^{n-v} + a_1 p^{n-v-1} + \dots + 1} \quad (4.4)$$

Where:

vis the static order of the system (number of ideal integration stages).

Transfer function in terms of bias:

$$W_E(p) = \frac{E(p)}{U(p)} = \frac{1}{1 + W(p)W_{ph}(p)}$$

$$E(p) = \frac{1}{1 + W(p)W_{ph}(p)} \cdot U(p) \quad (4.5)$$

According to the final limit property of the Laplace transform, expression (4.1) can be written as:

$$e_{xl} = \lim_{t \rightarrow \infty} e(t) = \lim_{p \rightarrow 0} p \cdot E(p) \quad (4.6)$$

Substituting the value of $E(p)$ into (4.6), we get the steady state error equal to:

$$e_{xl} = \lim_{t \rightarrow \infty} e(t) = \lim_{p \rightarrow 0} \frac{p}{1 + W(p)W_{ph}(p)} \cdot U(p) \quad (4.7)$$

$$e_{xl} = \lim_{p \rightarrow 0} \frac{p}{1 + \frac{K}{p^v} \cdot W_0(p)W_{ph}(p)} \cdot U(p) \quad (4.8)$$

Every time the system establishes a new steady state, the system will have a certain error that depends on the system's structure and impact. With a system working normally, it means that the structure of the system has not changed. Therefore, we only need to consider the establishment error e_{xl} with different inputs.

a) The input impact is a unit step function

The input action has the form $u(t) = C \cdot 1(t)$, with $C = \text{const}$.

The Laplace image of the function $C \cdot 1(t)$ is: $U(p) = L\{C \cdot 1(t)\} = C / p$

The system error is called static error or position error, denoted e_t .

From there, according to (4.10), we have:

$$e_t = \lim_{p \rightarrow 0} \frac{p}{1 + W_h(p)W_{ph}(p)} \cdot \frac{1}{p} = \frac{1}{1 + \lim_{p \rightarrow 0} W_h(p)W_{ph}(p)}$$

Set $K_p = \lim_{p \rightarrow 0} W_h(p)W_{ph}(p)$ - called the position coefficient.

We get the error according to position by: $e_t = \frac{1}{1 + K_p}$.

Consider the cases of static order v (ideal number of integration steps):

When $v=0$ (static system):

$$K_p = \lim_{p \rightarrow 0} W_h(p)W_{ph}(p) = \lim_{p \rightarrow 0} \frac{K}{p^0} W_0(p)W_{ph}(p) = \text{const}$$

Therefore: $e_t = \frac{C}{1 + K_p}$, this is called static error or position error.

When $v \geq 1$ (static system of order 1 or higher):

$$K_p = \lim_{p \rightarrow 0} W_h(p)W_{ph}(p) = \lim_{p \rightarrow 0} \frac{K}{p^1} W_0(p)W_{ph}(p) \rightarrow \infty$$

Therefore: $e_t = \frac{C}{1 + K_p} = \frac{C}{\infty} = 0$

We see that for a static (non-static) system, the static error is 0, so increasing the system's static level is also a measure to increase the system's accuracy. Figure 4.3 illustrates the static error when the input signal is a unit step function with different numbers of ideal integration stages.

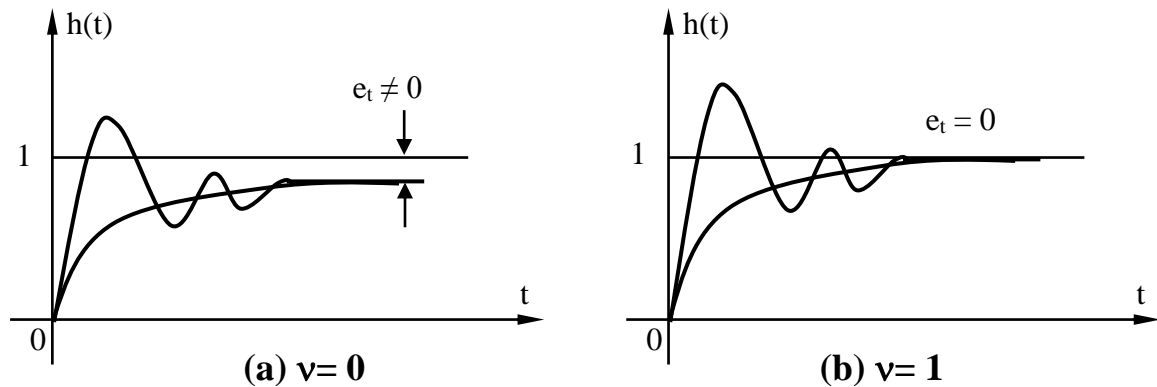


Figure 4.3. The error established with the input signal is a unit step
b) The input is a slowly changing function

The input action has the form $u(t) = Vt$, with $V = \text{const}$.

The Laplace image of the function $u(t)$ is:

$$U(p) = L\{Vt\} = \frac{V}{p^2}.$$

In the case that the input quantity is a function that changes slowly at a uniform rate $V = \text{const}$, the system error is called rate error or velocity and is denoted by e_v :

$$e_v = \lim_{p \rightarrow 0} \frac{p}{1 + W_h(p)W_{ph}(p)} \cdot \frac{V}{p^2} = \frac{V}{\lim_{p \rightarrow 0} pW_h(p)W_{ph}(p)}$$

Set $K_v = \lim_{p \rightarrow 0} pW_h(p)W_{ph}(p)$ - called the velocity coefficient.

We get the error according to speed by: $e_v = \frac{V}{K_v}$.

Consider the cases of static order:

When $v=0$ (the system is static, that is, there is no ideal integration stage in the system):

$$K_v = \lim_{p \rightarrow 0} pW_h(p)W_{ph}(p) = \lim_{p \rightarrow 0} p \frac{K}{p^0} W_0(p)W_{ph}(p) = 0$$

Therefore: $e_v = \frac{V}{K_v} = \frac{V}{0} \rightarrow \infty$, which means the error is very large and goes to infinity.

When $v = 1$ (first-order static system, that is, there is an ideal integration stage in the system):

$$K_v = \lim_{p \rightarrow 0} p W_h(p) W_{ph}(p) = \lim_{p \rightarrow 0} p \frac{K}{p^1} W_0(p) W_{ph}(p) = K = \text{const}$$

$$\text{Therefore: } e_v = \frac{V}{K_v} = \frac{V}{K} = \text{const.}$$

This error is proportional to the rate of change V of the input signal, so it is called speed error, and the transmission coefficient K of an open system is called the speed coefficient of the system. :

$$D = K$$

When $v \geq 2$ (static system of level 2 or higher):

$$K_v = \lim_{p \rightarrow 0} p W_h(p) W_{ph}(p) = \lim_{p \rightarrow 0} p \frac{K}{p^2} W_0(p) W_{ph}(p) \rightarrow \infty$$

$$\text{Therefore: } e_v = \frac{V}{K_v} = \frac{V}{\infty} = 0$$

With a static system the speed error is extremely large. For first-order static systems, the error with speed has a variable value. As for static systems of second order or higher, the speed error will be zero. Figure 4.4 illustrates the steady-state error when the input signal is a slowly changing function with different numbers of ideal integration stages, where e_v is the error between the input signal $u(t)$ and the output response $h(t)$ of the system.

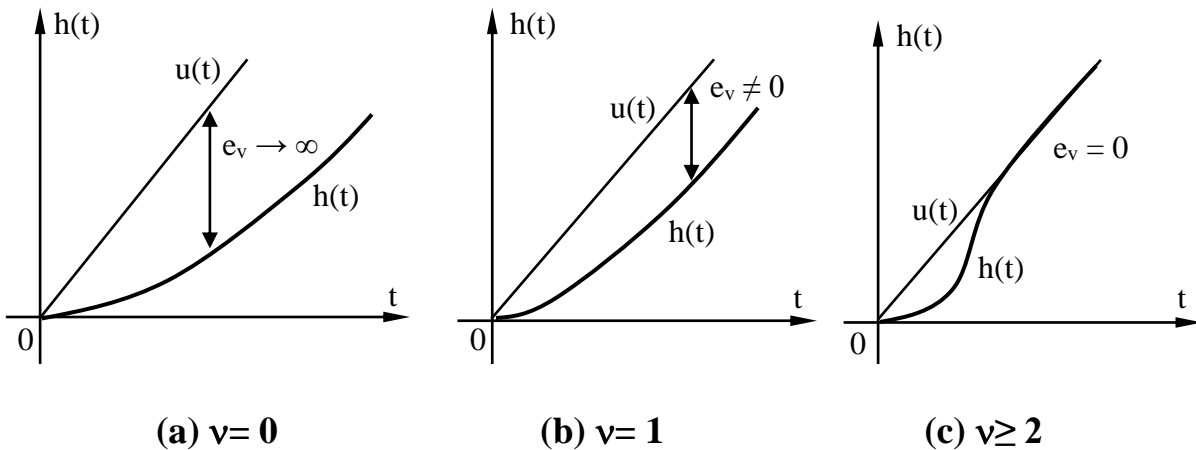


Figure 4.4. The steady-state error with the input signal is a slowly changing function

c) The input impact is a slowly changing function with acceleration

The input action has the form $u(t) = at^2$, with $a = \text{const.}$

The Laplace image of the function $u(t)$ is:

$$U(p) = L\{a.t^2\} = \frac{2a}{p^3}.$$

In the case that the input is a slowly changing function with acceleration $a = \text{const}$, the system error is called acceleration error and is denoted by e_a :

$$e_a = \lim_{p \rightarrow 0} \frac{p}{1 + W_h(p)W_{ph}(p)} \cdot \frac{2a}{p^3} = \frac{2a}{\lim_{p \rightarrow 0} p^2 W_h(p)W_{ph}(p)}$$

Set $K_a = \lim_{p \rightarrow 0} p^2 W_h(p)W_{ph}(p)$ - called acceleration coefficient.

We get the error according to acceleration by: $e_a = \frac{2a}{K_a}$.

Considering the cases of static order v , we have:

When $v = 0$ or 1 (first-order static and nonstatic systems): $e_a = \infty$

When $v = 2$ (2nd order static system): $e_a = \frac{2a}{K_a} = \text{const}$

When $v \geq 3$ (static system of order 3 or higher), then: $e_a = 0$.

Figure 4.5 illustrates the steady-state error (acceleration error) when the input signal is a slowly changing function of acceleration with different numbers of ideal integration stages.

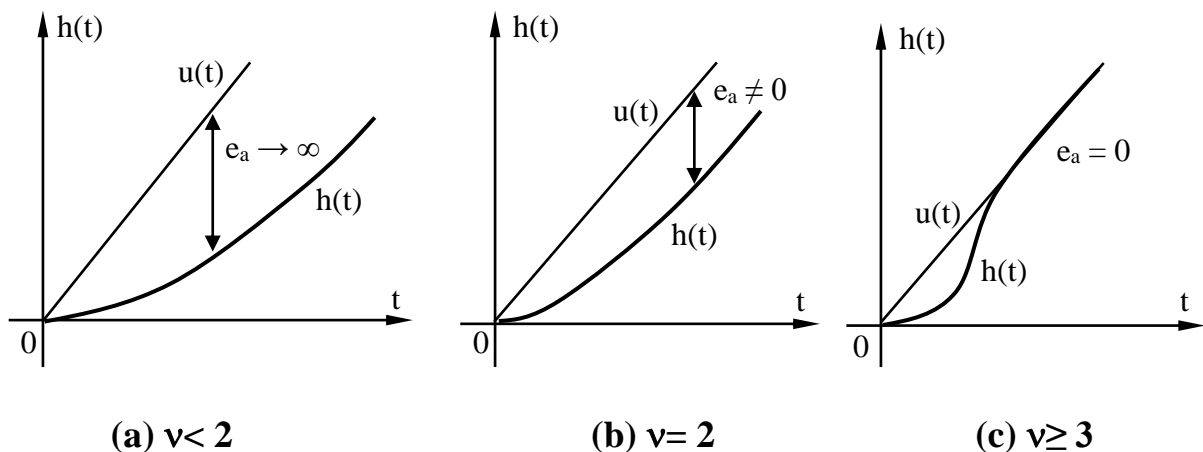


Figure 4.5. The steady-state error with the input signal is a slowly changing, accelerating function

d) The input impact is the sum of the component impacts

For a system whose input is the sum (or can be analyzed as a sum) of the component inputs, that is:

$$u(t) = C + Vt + at^2$$

Then the error is determined by the sum of the component errors:

$$e_{xl} = e_t + e_v + e_a$$

That is completely understandable because we use the superposition principle when examining linear systems.

Thus, we see that depending on the number of ideal integration stages in the transfer function $W_h(p)W_{ph}(p)$, the coefficients K_p , K_v , K_a have different values, as shown. presented in table 4.1.

Table 4.1. The relationship between the number of stages in the establishment process and the coefficients

vin $W_h(p)W_{ph}(p)$	Position coefficient K_p	Speed coefficient K_v	Acceleration coefficient K_a
0	$K_p = \text{const}$	0	0
1	∞	$K_v = \text{const}$	0
2	∞	∞	$K_a = \text{const}$
≥ 3	∞	∞	∞

Comment:

If the system's steady-state error for the input signal is a unit step function of zero, the transfer function $W_h(p)W_{ph}(p)$ must have at least one ideal integration stage.

If the system's steady-state error for the input signal is a slowly changing function equal to 0, the transfer function $W_h(p)W_{ph}(p)$ must have at least two ideal integration stages.

If the system's steady-state error for the input signal is a slowly changing function with zero acceleration, the transfer function $W_h(p)W_{ph}(p)$ must have at least three ideal integration stages.

Example 4.1.1. Determine the steady-state error of the system with the structural diagram depicted in Figure 4.6 with input signals $u = 15$ and $u = 3t$.

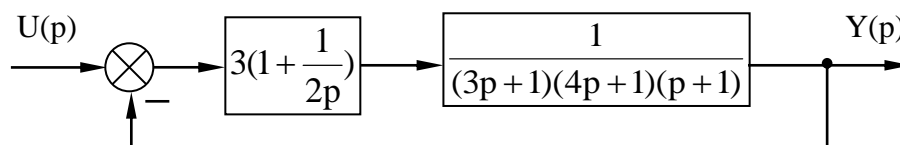


Figure 4.6. Unit negative feedback system

The answer:

Transfer function of open system:

$$W(p) = \frac{3(2p+1)}{2p(3p+1)(4p+1)(p+1)}$$

It can be analyzed as follows:

$$W(p) = \frac{1,5}{p} \cdot \frac{2p+1}{12p^3 + 19p^2 + 8p + 1}$$

Transmission coefficient of the system: $K = 1.5$.

Number of integration stages in the system: $v = 1$.

According to the analysis results in the above sections, we can conclude:

- When the input is $u = 15$, the system's static error is $e_t = 0$;
- When the impact is $u = 3t$, the error according to the system's velocity $e_v = 3/1,5 = 2$.

4.1.3. Error coefficient method

a) Error coefficient

In practice, the impact of slow change is often encountered, but the components cannot be separated as considered above. To determine the steady-state error in this case, we introduce the concept of error coefficient.

How to determine the system's steady-state error can be carried out as follows:

We have a related expression

$$W_E(p) = \frac{E(p)}{U(p)} = \frac{B(p)}{A(p)} \quad (4.9)$$

In which:

$E(p)$ is the error in the establishment process written in operator form,

$W_E(p)$ is the error transfer function written as an operator,

$U(p)$ is the operation written as an operator.

Here the polynomials numerator $B(p)$ and denominator $A(p)$ have the same degree. Finding the steady-state error means finding the error at time $t \rightarrow \infty$ or $p \rightarrow 0$. That means p is very small, so we can implement the transfer function $W_E(p)$ into a Macloranh series:

$$W_E(p) = \frac{b_0p^m + b_1p^{m-1} + \dots + b_m}{a_0p^n + a_1p^{n-1} + \dots + a_n} = C_0 + C_1p + C_2p^2 + \dots + C_ip^i \quad (4.10)$$

In which: $i = 0, 1, \dots n$.

Therefore, the error signal has the form:

$$E(p) = W_E(p) \cdot U(p) = C_0 U(p) + C_1 p U(p) + C_2 p^2 U(p) + \dots + C_i p^i U(p) \quad (4.11)$$

Or switch back to the original function:

$$e(t) = C_0 u(t) + C_1 \frac{du(t)}{dt} + C_2 \frac{d^2 u(t)}{dt^2} + \dots + C_i \frac{d^i u(t)}{dt^i} \quad (4.12)$$

Or
$$e(t) = e_t + e_v + e_a + \dots$$

In there:

$$e_t = C_0 u(t) \quad \text{- Position error (static error);}$$

$$e_v = C_1 \frac{du(t)}{dt} \quad \text{- Dynamic error according to speed;}$$

$$e_a = C_2 \frac{d^2 u(t)}{dt^2} \quad \text{- Dynamic error according to acceleration.}$$

Similarly, we have dynamic errors according to the higher derivative of the input signal.

Expression (4.15) is the error established over time.

Coefficients C_i ($i = 0, 1, \dots, n$) is called the error coefficient. With the input impact known, we only need to find the error coefficients to determine the steady-state error.

b) Methods for determining error coefficients

Determine the error coefficient C_i by many methods such as direct calculation, polynomial division method, coefficient balancing method... In practice, the coefficient balancing method is often used.

- Direct method of calculating C_i

Use the formula to directly calculate the coefficients C_i as follows:

$$C_i = \frac{1}{i!} \lim_{p \rightarrow 0} \frac{d^i W_E(p)}{dp^i} \quad (4.13)$$

In which: $i = 0, 1, \dots, n$.

- Coefficient balancing method

From the Macloranh series approximation (4.10):

$$W_E(p) = \frac{b_0 p^m + b_1 p^{m-1} + \dots + b_m}{a_0 p^n + a_1 p^{n-1} + \dots + a_n} = C_0 + C_1 p + \dots + C_i p^i$$

We cross-multiply both sides, group terms of the same order, then set up a system of equations to solve for the coefficients C_i .

For example, with $n = 3$ we find the following error coefficients:

$$\begin{aligned}
 C_0 &= \frac{d_3}{a_3} \\
 C_1 &= \frac{1}{a_3} (d_2 - c_0 a_2) \\
 C_2 &= \frac{1}{a_3} (d_1 - c_1 a_0 - c_1 a_2) \\
 C_3 &= \frac{1}{a_3} (d_0 - c_0 a_0 - c_1 a_1 - c_2 a_2).
 \end{aligned}
 \tag{4.14}$$

Example 4.1.3. Calculate the steady-state error of a unit negative feedback system whose open system transfer function is

$$W(p) = \frac{100 \cdot (0,5p + 1)}{p \cdot (0,008p^3 + 0,1p^2 + 2,05p + 1)}$$

Know the impact on $u(t) = 3 + 6t + 9t^2$.

The answer:

Method 1: Direct calculation method C_i

We have the transfer function in terms of error equal to:

$$W_E(p) = \frac{E(p)}{U(p)} = \frac{1}{1 + W(p)} = \frac{p \cdot (0,008p^3 + 0,1p^2 + 2,05p + 1)}{p \cdot (0,008p^3 + 0,1p^2 + 2,05p + 1) + 100 \cdot (0,5p + 1)}$$

To summarize, we get:

$$W_E(p) = \frac{0,008p^4 + 0,1p^3 + 2,05p^2 + p}{0,008p^4 + 0,1p^3 + 2,05p^2 + 51p + 100}$$

Use formula (4.16) to calculate coefficients C_i :

$$C_0 = \lim_{p \rightarrow 0} W_E(p) = 0$$

$$C_1 = \lim_{p \rightarrow 0} \frac{dW_E(p)}{dp} = 0,01$$

$$C_2 = \frac{1}{2!} \lim_{p \rightarrow 0} \frac{d^2 W_E(p)}{dp^2} = 0,0154$$

So according to (4.15) the error is set by:

$$e(t) = 0 \cdot u(t) + 0,01 \cdot \frac{du(t)}{dt} + 0,0154 \cdot \frac{d^2 u(t)}{dt^2}$$

$$e(t) = 0,01 \cdot (6 + 18t) + 0,0154 \cdot 18 = 0,18t + 0,3372.$$

Method 2: Coefficient balancing method

We have an expression to approximate the transfer function $W_E(p)$ into a Macloranh series as

$$W_E(p) = \frac{0,008p^4 + 0,1p^3 + 2,05p^2 + p}{0,008p^4 + 0,1p^3 + 2,05p^2 + 51p + 100} = C_0 + C_1p + C_2p^2 + C_3p^3 + \dots$$

Cross multiplying the two sides, grouping terms of the same order, we can create the following system of equations:

$$\begin{cases} 0 = 100.C_0 \\ 1 = 51.C_0 + 100C_1 \\ 2,05 = 2,05.C_0 + 51.C_1 + 100.C_2 \end{cases}$$

Here we only need to find the coefficient C_2 , because the third or higher derivative of the input impact is zero.

Solving the above system of equations we get:

$$\begin{cases} C_0 = 0 \\ C_1 = 0,01 \\ C_2 = 0,0154 \end{cases}$$

Similar to method 1, using (4.15) we can calculate the setup error by:

$$e(t) = 0 + 0,01.(6 + 18t) + 0,0154.18 = 0,18t + 0,3372.$$

4.1.4. Methods to improve the quality of automatic control systems during the setup process

a) Increase the level of tranquility

Since the established error depends on the input, the derivatives of the input and depend on the error coefficient:

$$e(t) = C_0u(t) + C_1 \frac{du(t)}{dt} + C_2 \frac{d^2u(t)}{dt^2} + \dots + C_i \frac{d^i u(t)}{dt^i} \quad (4.15)$$

In which $u(t)$ is the control signal rule, so this rule cannot be removed or changed. So to reduce the steady-state error, just by reducing the error coefficients C_i .

From (4.18), we see that if we want to eliminate the error up to the $n-1$ derivative level, we must raise the static order of the system to n .

When the static order $v = 0$, then $C_0 \neq 0$, there exists a set error value from the position error onwards:

$$e(t) = C_0u(t) + C_1 \frac{du(t)}{dt} + \dots$$

When $\nu = 1$, then $C_0 = 0$, $C_1 \neq 0$, so there is only one set error value calculated from the error according to velocity (first derivative) onwards:

$$e(t) = C_1 \frac{du(t)}{dt} + C_2 \frac{d^2u(t)}{dt^2} + \dots$$

When $\nu = 2$, then $C_0 = C_1 = 0$, $C_2 \neq 0$, so there is only one set error value calculated from the error according to acceleration (second derivative) onwards:

$$e(t) = C_2 \frac{d^2u(t)}{dt^2} + C_3 \frac{d^3u(t)}{dt^3} + \dots$$

Thus, we see that raising the static level of the system can reduce the value of the stability error, but raising the static level can also cause instability of the system.

b) Increase the amplification factor

According to the analysis in the above sections, we see that the error coefficients are inversely proportional to the amplification (transformation) coefficient K of the system. Therefore, increasing the amplification factor will reduce the error in the system (derived from the formula e_{xl}). But this method increases the system's cutoff frequency, leading to a decrease in system stability, and can even make the system unstable. It is up to each system to increase the amplification factor to a reasonable value (critical value) K_{th} .

c) Additional additional steps are required

When the structure changes, the dynamic properties of the system change, changing the quality of the system. To change the structure, we can add a system of supporting stages. Choosing the structure and parameters of the supporting stage turns an initial system that does not meet quality requirements into a system that ensures the quality requirements set for it.

There are three ways to add additional stages to the system to improve the quality of the control system during the setup process: series connection, parallel connection and mixed connection.

- Connected in series

This way of connecting is to put the auxiliary link in series with the links in the main circuit of the system. Or in other words, adding a series connection to the open system transfer function as illustrated in Figure 4.7.

We can use the differential or integral step as a complementary step.

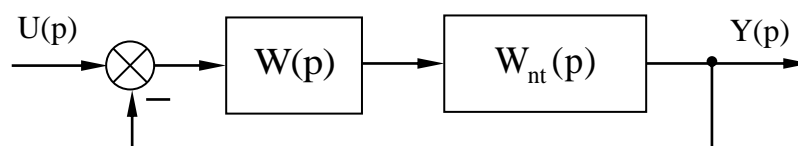


Figure 4.7. System with auxiliary connection connected in series

The advantage of this method is that the system is simple. But it does not directly improve the characteristics of individual stages, which negatively affects the dynamic quality of the system, and is also sensitive to noise.

- Connected in parallel (secondary feedback wire)

This way of connecting is to introduce a supporting stage into the system by connecting in an auxiliary feedback style that covers one or a group of elements in the main circuit.

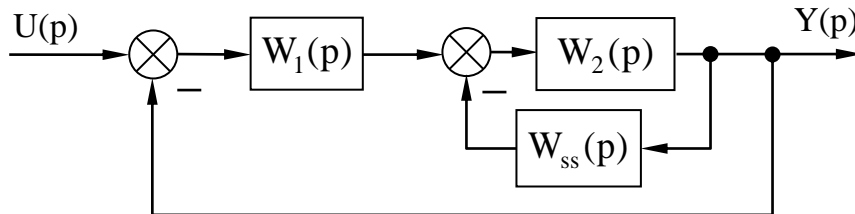


Figure 4.8. System with auxiliary connections connected in parallel

Often people make the supporting stage cover the structure, which has a negative effect on the dynamic quality of the system as illustrated in Figure 4.8.

The advantage of this method is that when selecting auxiliary structures, there is no need to satisfy strict quality conditions, because some included structures have little effect on the dynamic properties of the system. Furthermore, this mechanism is less sensitive to noise, thus reducing random errors.

However, this method has disadvantages: The auxiliary stage is complex and cumbersome, and the auxiliary stage often requires a large amplification coefficient and complex correction calculations.

- Mixed hooks

Combining the above two methods (Figure 4.9), the disadvantages of the two methods can be overcome to improve quality.

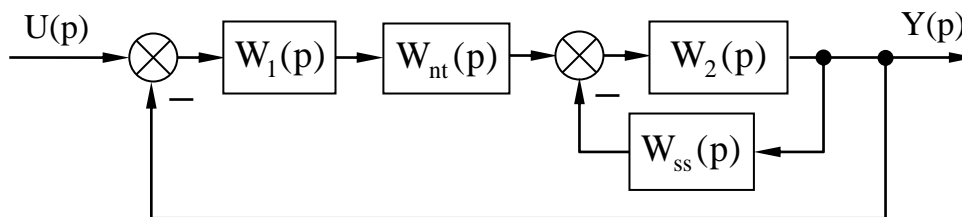


Figure 4.9. System with mixed auxiliary stages

Each method of improving system quality in the establishment process has its own advantages and disadvantages, so it has a certain scope of application. Depending on the actual problem, the ability of the synthesizer and the calculation tool, choose the most appropriate method to achieve high efficiency.

4.2. EVALUATING THE QUALITY OF THE SYSTEM DURING THE TRANSITION PROCESS

4.2.1. Quality criteria in the transition process

An automatic control system is said to be stable when the system's output signal gradually decreases over time:

$$\lim_{t \rightarrow \infty} y_{qd}(t) = 0 \quad (4.16)$$

Or the output signal of the system when the input signal $u(t)$ is a unit step function $1(t)$ will approach a constant stable value. The curve in Figure 4.1 can fluctuate around the value h_{xl} . Above and below the value of h_{xl} , we take 5% of h_{xl} and draw it into a strip. If the curve $h(t)$ falls within the band $\pm 5\% h_{xl}$, suppose it cuts this band at point G. At point G, we can determine the system's transient time as T_{qd} . The indicators to evaluate the quality of the transition process are summarized as follows:

- Transition time T_{qd}

Determined by the length of the transition process. In theory, the transition process lasts indefinitely but in reality it is considered to end when the deviation of the controlled quantity from its new set value does not exceed the allowable limit of Δ .

Usually Δ choose equal to $(2 \div 5)\%$ compared to h_{xl} . People characterize the rapidity of the system by its transient time. But there are also documents that characterize it by the time T_m when the transition function first reaches a new set value or by the time with an adjustment T_δ when the transition function reaches its maximum value h_{max} .

- Adjustable overshoot δ

Over-adjustment is the maximum deviation of the amount to be adjusted compared to the new set value.

The relative overcorrection is calculated by

$$\delta = \frac{h_{max} - h_{xl}}{h_{xl}} \cdot 100\% \quad (4.2)$$

Overshoot occurs because the system approaches a new steady state at a certain speed. This speed is equal to the angle of inclination of the tangent line at point A:

$$\text{tg}\alpha = \left. \frac{dh(t)}{dt} \right|_{t=T} \quad (4.3)$$

The greater this speed is, the further the system, due to inertia, moves away from the new set position. Thus, if you want to reduce over-regulation, you need to reduce the speed of the system, that is, reduce the rapidity of action.

But people always want to reduce the transient time, which means processing the control signal at a high speed leads to a large overshoot. Therefore, the overcorrection must be selected at an appropriate value. In practice, choosing an overcorrection level within the range of (20÷30)% is often optimal.

Adjustment overshoot and transition time are two basic indicators of the transition process. People always want both of those criteria to be the smallest, because the smaller T_{qd} , the faster the system acts and the smaller δ , the easier the system is to realize. That wish cannot be realized, because they have opposing properties, that is, if the adjustment period is small, the transition time is large and vice versa. To ensure the quality of the system (in some optimal sense), two appropriate criteria must be selected, and they should not be separated during the survey process.

In addition to the two basic criteria above, there are also:

Response time T_m : Determined by the time when the transient function first reaches the set value h_{xl} when there is an overshoot.

Overshoot time T_δ : Determined by the time the transient function reaches its maximum value.

Number of oscillations N : Determined by the number of times the controlled quantity oscillates around the new set value during the transition time $N = T_{qd}/T_0$ (with $T_0 = 2\pi\omega_0$ being the oscillation period natural movement). In fact, people often choose $N = 1\div 2$.

When evaluating the quality of the system during the transition process, the criteria δ and N characterize the accuracy of the transition process; while T_{qd} , T_δ , T_m characterize the fast-acting nature of the system.

4.2.2. Transient response analysis

a) *Transient response of a first-order inertial system*

- Overview of first-order inertial systems

According to expression (2.37), we have the function that conveys the first-order inertial system as:

$$W(p) = \frac{K}{Tp + 1} \quad (4.19)$$

A first-order inertial system has one real pole: $p_1 = -\frac{1}{T}$.

The transient response of a first-order inertial system is equal to:

$$h(t) = L^{-1}\{U(p).W(p)\} = L^{-1}\left\{\frac{1}{p} \cdot \frac{K}{Tp + 1}\right\} = K(1 - e^{-t/T}) \quad (4.20)$$

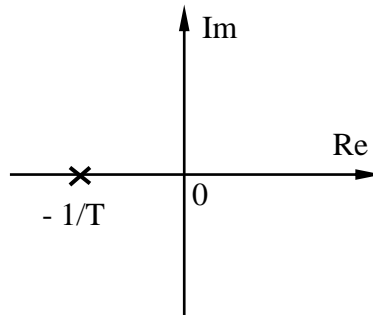


Figure 4.10. Pole-zero diagram of first-order inertial phase

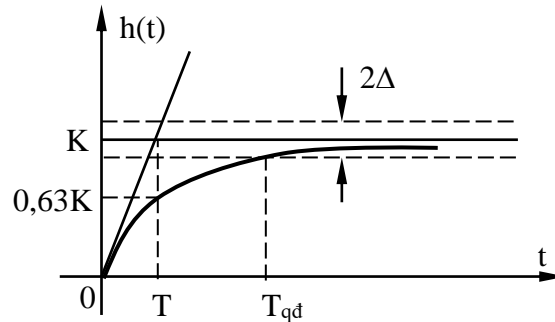


Figure 4.11. Transient response of first-order inertial phase

- Comments on first-order inertial systems

The first-order inertial system has only one real pole $p_1 = -1/T$ (Figure 4.10), the transient response has no overshoot as described in Figure 4.11.

The time constant T is determined by the time the response of the first-order inertial link reaches 63% of the established value.

The transient time of a first-order inertial system is: $t_{qd} = T \ln\left(\frac{1}{\Delta}\right)$

With $\Delta = 0.02$ (2% standard) or $\Delta = 0.05$ (5% standard).

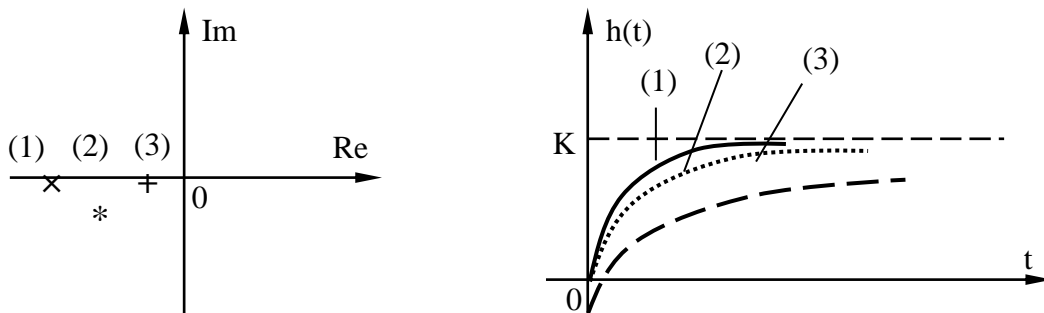


Figure 4.12. Relationship between pole position and transient response

The farther the real pole ($-1/T$) is from the imaginary axis, the smaller the time constant T , the faster the system responds, and the shorter the transient time, as illustrated in Figure 4.12.

b) Transient response of a second-order oscillation system

- Overview of second-order oscillation systems

According to expression (2.46), we have the transfer function of the second-order oscillation system as:

$$W(p) = \frac{K}{T^2 p^2 + 2\xi T p + 1} = \frac{K\omega_n^2}{p^2 + 2\xi\omega_n p + \omega_n^2} \quad (4.21)$$

In there:

$$\omega_n = \frac{1}{T} - \text{Natural oscillation frequency,}$$

$$0 < \xi < 1 - \text{Damping coefficient.}$$

The second-order oscillator system has a pair of complex poles:

$$p_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} \quad (4.22)$$

The transient response of a second-order oscillation system is equal to:

$$\begin{aligned} h(t) &= L^{-1}\{U(p)W(p)\} = L^{-1}\left\{\frac{1}{p} \cdot \frac{K\omega_n^2}{p^2 + 2\xi\omega_n p + \omega_n^2}\right\} = \\ &= K \left\{1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left[(\omega_n\sqrt{1-\xi^2})t + \theta\right]\right\} \end{aligned} \quad (4.23)$$

With $\cos \theta = \xi$.

The pole-zero point diagram and transient response of the second-order oscillation system are depicted in Figures 4.13 and 4.14.

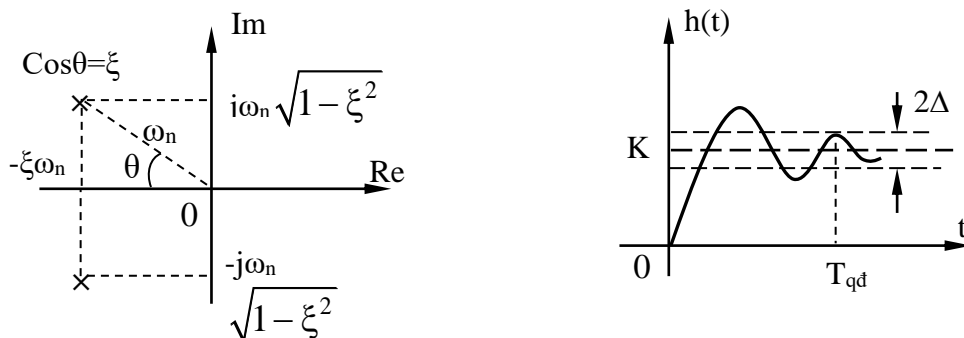


Figure 4.13. Pole-zero diagram of Figure 4.14. Transient response of quadratic oscillation stage quadratic oscillation stage

- Comments on second-order oscillation systems

The second-order oscillation system has a complex pair of poles, the transient response has the form of oscillation with decreasing amplitude, in which:

If $\xi = 0$, the response of the system is undamped oscillation with frequency ω_n , then ω_n is called the natural oscillation frequency.

If $0 < \xi < 1$, the response of the system is to oscillate with decreasing amplitude, ξ is called the damping coefficient (or attenuation coefficient), the larger ξ is (the closer the pole is to the real axis), the more attenuated the oscillation is. decrease faster.

Transient response of a second-order oscillator system with overcorrection:

$$\delta = e^{\left(-\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)}.100\% \quad (4.24)$$

According (4.24), we see that the larger ξ (the closer the pair of poles is to the real axis), δ the smaller it is, the smaller ξ (the closer the pair of complex poles is to the imaginary axis), δ the larger it is. The relationship between damping coefficient and overshoot is depicted in Figure 4.15.

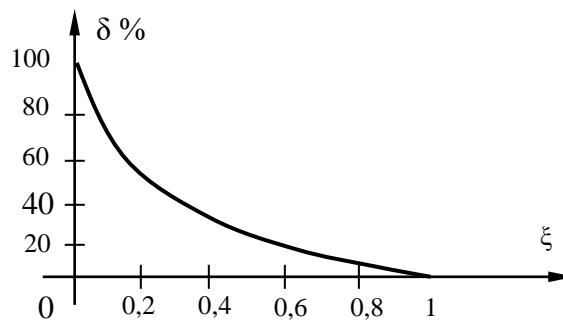


Figure 4.15. Relationship between damping coefficient and overcorrection

The transient time of a second-order oscillation system is calculated in two cases:

According to the 5% standard:

$$t_{qd} = \frac{3}{\xi\omega_n} \quad (4.25)$$

According to the 2% standard:

$$t_{qd} = \frac{4}{\xi\omega_n} \quad (4.26)$$

- Relationship between pole position and transient response of second-order oscillation system

Second-order oscillation systems with poles located on the same ray originating from the origin have equal damping coefficients, and therefore have equal overcorrections. Any system whose pole is farther from the origin has a larger natural oscillation frequency, so the transient time is shorter. This relationship is depicted in Figure 4.16.

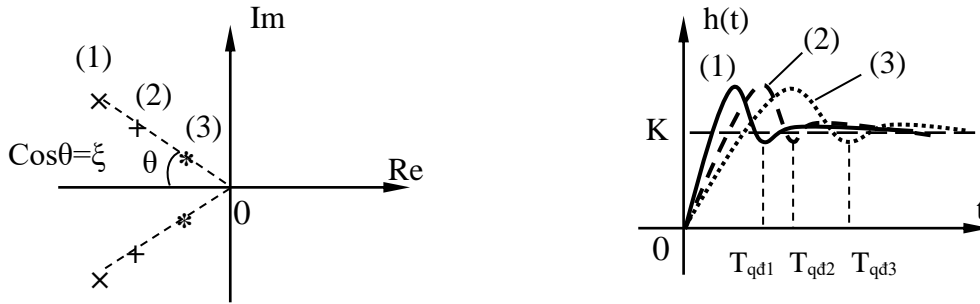


Figure 4.16. The pole lies on the same ray originating from the origin

Second-order oscillating systems with poles located at an equal distance from the origin have the same natural oscillation frequency; systems with poles located closer to the virtual axis have a smaller damping coefficient, so the transient adjustment is larger, the transition time is longer. This relationship is depicted in Figure 4.17.

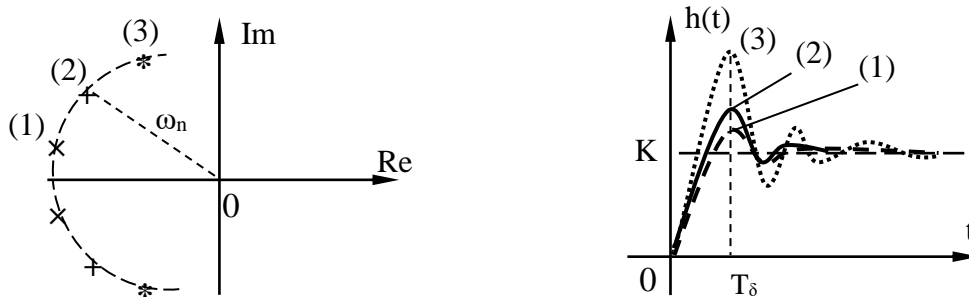


Figure 4.17. The pole is located at an equal distance from the origin

Second-order oscillation systems with poles located at an equal distance from the imaginary axis have equal $\xi\omega_n$, so the transient times are equal. Any system whose pole is farther from the real axis has a smaller damping coefficient, so the overcorrection is higher. This relationship is depicted in Figure 4.18.

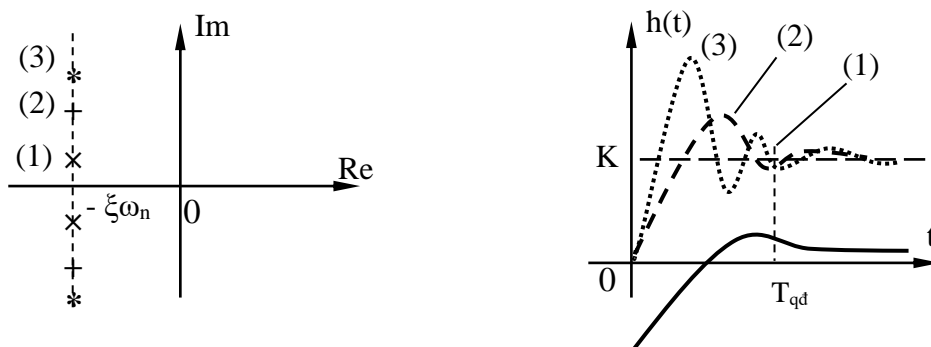


Figure 4.18. The pole is located at an equal distance from the virtual axis

4.2.2. Integral standards

Evaluating system quality according to integral criteria means using a definite integral of a certain function as a basis to evaluate the quality. For automatic control systems, the error function $e(t)$ is usually taken.

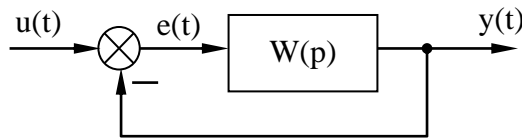


Figure 4.19. Unit negative feedback system

Suppose a system is shown in Figure 4.19, with the input being a unit step function:

$$e(t) = u(t) - y(t) = 1(t) - h(t) \quad (4.27)$$

a) Simple integration

As we know, the transition function has two types: non-oscillating characteristics and oscillatory characteristics, corresponding to that the error function also has two types as depicted in Figure 4.20.

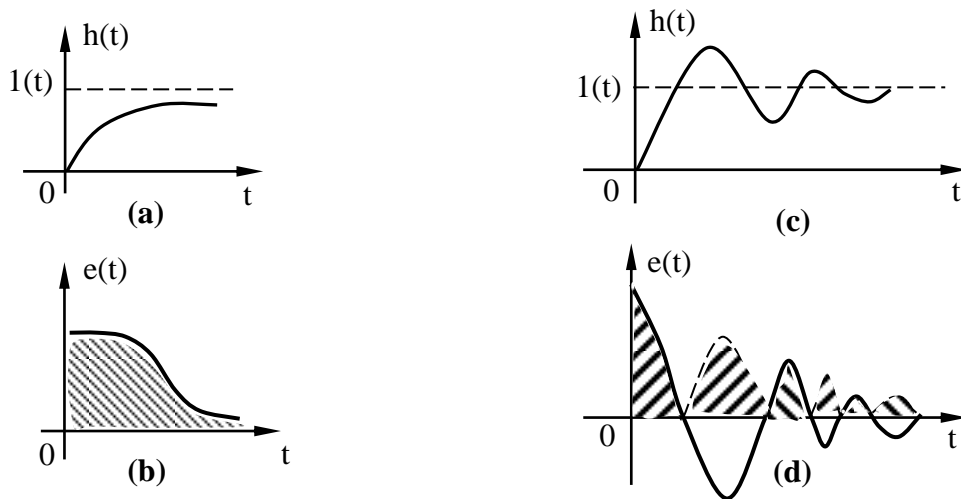


Figure 4.20. Represents transient characteristics and error functions

If the transient characteristic $h(t)$ is a monotonic process as shown in Figure 4.20a, then the corresponding error function $e(t)$ is also a monotonic function as shown in Figure 4.20b.

If the transient characteristic $h(t)$ is an oscillating process as shown in Figure 4.20c, then the corresponding error function $e(t)$ has the form as shown in Figure 4.20d.

We see that the area created by the line $e(t)$ and the horizontal axis (striped part) is proportional to the correction degree δ and the transition time T_{qd} , that area is calculated by the following integration:

$$I_1 = \int_0^{\infty} e(t)dt \quad (4.28)$$

If the quality of the system is good, I_1 must reach the smallest possible value.

So when using integration to evaluate the quality of the system, it is called quality integration.

How to calculate integral I_1 :

According to generalized integration we have

$$I_1 = \int_0^{\infty} e(t)dt = \lim_{t \rightarrow \infty} \int_0^t e(t)dt = \lim_{p \rightarrow 0} \int_0^t e(t).e^{-pt} dt$$

According to the forward Laplace transform

$$E(p) = \int_0^{\infty} e(t).e^{-p.t} dt$$

So we get it

$$I_1 = \lim_{p \rightarrow 0} E(p) \tag{4.29}$$

So instead of calculating I_1 according to (4.28), we just calculate I_1 according to the simpler expression (4.29).

For oscillatory transients, the area according to (4.28) can be very small but the correction degree and transient time can be large, because the two areas, negative and positive, offset each other, so use (4.28) to evaluate the oscillatory transient process is not appropriate. Therefore, the integral I_1 can only be used to evaluate the non-oscillating transient process .

To overcome this problem, we propose the following integral criteria:

$$I_2 = \int_0^{\infty} |e(t)| dt \tag{4.30}$$

With this formula, the sign of $e(t)$ no longer affects the value of the integral. Although I_2 can evaluate the quality of the oscillating or non-oscillating transient process , in reality it is rarely used because to calculate according to formula (4.30), the variation path of $e(t)$ must be known in advance.

b) Integral squared error

We see that calculating I_2 is difficult, because the function under the integral sign is an absolute value, so people give the square integral of the error function:

$$I_3 = \int_0^{\infty} e^2(t) dt \tag{4.31}$$

The way to calculate the estimate of the squared error integral I_3 is as follows:

Substitute $p = j\omega$ into the forward Laplace transform

$$E(j\omega) = \int_0^{\infty} e(t).e^{-j\omega t} dt \quad (4.32)$$

and inverse transformation

$$e(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E(j\omega).e^{j\omega t} d\omega \quad (4.33)$$

Substituting (4.31) and (4.32) into (4.30) and transforming, we get

$$\begin{aligned} I_3 &= \int_0^{\infty} e^2(t)dt = \int_0^{\infty} e(t).e(t)dt = \int_0^{\infty} e(t) \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} E(j\omega).e^{j\omega t} d\omega \right] dt = \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} E(j\omega) \left[\int_0^{\infty} e(t).e^{j\omega t} dt \right] d\omega \end{aligned}$$

Let $E^*(j\omega) = \int_0^{\infty} e(t).e^{j\omega t} dt$ be the complex conjugate expression of $E(j\omega)$, so

$$I_3 = \frac{1}{2\pi} \int_{-\infty}^{\infty} E(j\omega).E^*(j\omega)d\omega$$

So we can

$$I_3 = \frac{1}{2\pi} \int_{-\infty}^{\infty} |E(j\omega)|^2 d\omega \quad (4.34)$$

Where the expression for the bias function is equal to

$$E(j\omega) = \frac{b_0(j\omega)^{n-1} + b_1(j\omega)^{n-2} + \dots + b_m}{a_0(j\omega)^n + a_1(j\omega)^{n-1} + \dots + a_n} = \frac{B(j\omega)}{A(j\omega)}$$

Expression (4.34) is the Parseval integral for calculating I_3 and the optimal parameters of the system according to I_3 .

We often calculate I_3 using available formulas (replace $j\omega = p$):

$$E(p) = \frac{b_0p^m + b_1p^{m-1} + \dots + b_{m-1}p + b_m}{a_0p^n + a_1p^{n-1} + \dots + a_{n-1}p + a_n}$$

The degree in the numerator is smaller than the degree in the denominator ($m < n$).

If $m = n - 1$, we can calculate I_3 with the following formula:

$$E(p) = \frac{b_0}{a_0p + a_1} \Rightarrow I_3 = \frac{b_0^2}{2a_0a_1} \quad (4.35)$$

$$E(p) = \frac{b_0p + b_1}{a_0p^2 + a_1p + a_2} \Rightarrow I_3 = \frac{b_1^2a_0 + b_0^2a_2}{2a_0a_1a_2} \quad (4.36)$$

$$E(p) = \frac{b_0p^2 + b_1p + b_2}{a_0p^3 + a_1p^2 + a_2p + a_3} \Rightarrow I_3 = \frac{b_2^2a_0a_1 + (b_1^2 - 2b_0b_1)a_0a_3 + b_0^2a_2a_3}{2a_0a_3(a_1a_2 - a_0a_3)} \quad (4.37)$$

c) *Advanced squared error integral*

In many cases, the I_3 standard does not accurately represent the quality of the transition process. For example, in Figure 4.21, the system corresponding to line $e_2(t)$ has better quality than the system corresponding to line $e_1(t)$, because $e_2(t)$ is a characteristic line that does not fluctuate. But when calculating the integral I_3 , the evaluation of the integral corresponding to the line $e_1(t)$ is smaller than the evaluation of the integral corresponding to the line $e_2(t)$.

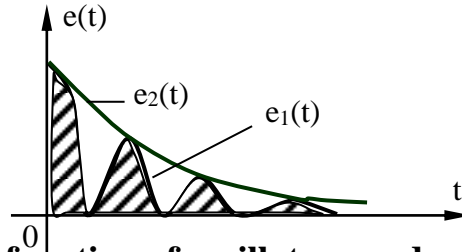


Figure 4.21. Deviation function of oscillatory and non-oscillating transients

Or in other words, if calculated according to (4.31), it does not accurately represent the quality of the transition process (the fluctuating nature of the deviation). Therefore, the component $de(t)/dt$ is included to reflect the rate of change and fluctuation of $e(t)$ as follows:

$$I_4 = \int_0^{\infty} [e^2(t) + \tau^2 \left(\frac{de}{dt}\right)^2] dt \quad (4.38)$$

τ^2 is a fixed value, usually τ^2 is chosen in the range:

$$\frac{T_{qd}}{6} < \tau^2 < \frac{T_{qd}}{3}$$

Integrating I_4 gives us a complete assessment of the quality of the transition process. When I_4 reaches the smallest value, it means that a small I_3 is achieved but the rate of change of the error is not high. For each individual system, an appropriate value of τ^2 must be chosen. You can choose a small τ^2 with a transition process that allows large fluctuations.

How to estimate integral I_4 :

We have

$$I_4 = \int_0^{\infty} e^2(t)dt + \tau^2 \int_0^{\infty} \left(\frac{de}{dt}\right)^2 dt = I_3 + \tau^2 \cdot I_3^* \quad (4.39)$$

In there

$$I_3^* = \int_0^{\infty} \left(\frac{de}{dt}\right)^2 dt \quad (4.40)$$

To calculate, I_3^* we set $m(t) = \frac{de(t)}{dt}$, perform Laplace transform

$$M(p) = pE(p) - e(0)$$

According to the initial value theorem $e(0) = \lim_{t \rightarrow 0} e(t) = \lim_{p \rightarrow \infty} pE(p)$

Therefore

$$M(p) = pE(p) - \lim_{p \rightarrow \infty} pE(p) \quad (4.41)$$

So in expression (4.39) we have to calculate I_3 and I_3^* , which I_3^* can be calculated from the expression $M(p)$ using the available formula like calculating I_3 .

Example 4.2.1. Given a system with unit negative feedback, know the open system transfer function is $W(p) = \frac{K}{p(Tp+1)}$. In which $K, T > 0$ and $\tau=0,5$.

Determine quality integrals.

The answer:

To evaluate the quality of the system's transition process, the impact must be a function of $1(t)$.

The error function:

$$E(p) = \frac{1}{1+W(p)} \cdot U(p) = \frac{Tp+1}{Tp^2+p+K}$$

According to the quality integral expression (4.29), we can calculate:

$$I_1 = \lim_{p \rightarrow 0} E(p) = \lim_{p \rightarrow 0} \frac{Tp+1}{Tp^2+p+K} = \frac{1}{K}$$

The squared error integral I_3 is calculated using the formula:

$$I_3 = \frac{b_1^2 a_0 + b_0^2 a_2}{2a_0 a_1 a_2} = \frac{1+KT}{2K}$$

To calculate the advanced squared difference integral I_4 , we calculate according to (4.41).

$$M(p) = pE(p) - \lim_{p \rightarrow \infty} pE(p) = \frac{-K}{Tp^2 + p + K}$$

Then I_3^* equals

$$I_3^* = \frac{b_1^2 a_0 + b_0^2 a_2}{2a_0 a_1 a_2} = \frac{K}{2}$$

Therefore, we can calculate the quality integral I_4 as:

$$I_4 = I_3 + \tau^2 \cdot I_3^* = \frac{1}{2} \left(T + \frac{1}{K} + 0,5^2 \right).$$

4.3. CHAPTER 4 EXERCISES

4.3.1. Sample solution exercises

Exercise 1. Calculate the steady-state error of a system with a structural diagram as shown in Figure 4.22.

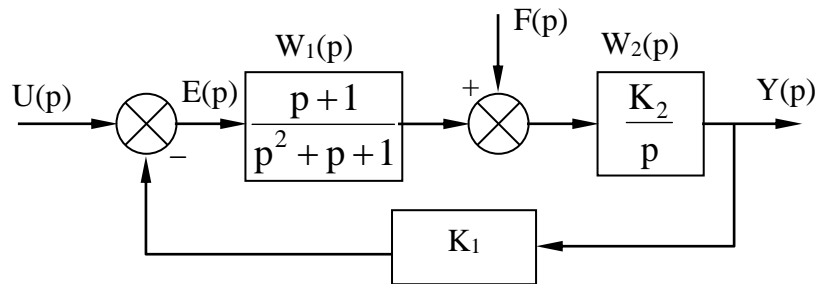


Figure 4.22. The automatic control system has noise

In there: $u(t) = \begin{cases} 1 & \text{when } t \geq 0 \\ 0 & \text{when } t < 0 \end{cases}$; and noise $f(t) = A\delta(t)$.

The answer:

We know: $Y(p) = W_1(p)W_2(p)E(p) + W_2(p)F(p)$

On the other hand: $E(p) = U(p) - K_1 Y(p)$

$$\Rightarrow Y(p) = W_1(p)W_2(p) \cdot [U(p) - K_1 Y(p)] + W_2(p)F(p)$$

Transforming we get:

$$Y(p) = \underbrace{\frac{W_1(p)W_2(p)}{1+W_1(p)W_2(p)K_1}}_{Y_U(p)} U(p) + \underbrace{\frac{W_2(p)}{1+W_1(p)W_2(p)K_1}}_{Y_F(p)} F(p)$$

The system has two inputs, so the error function is the sum of the component errors:

$$E(p) = E_U(p) + E_F(p)$$

In there:

$$E_U(p) = U(p) - Y_U(p) = \left[1 - \frac{W_1(p)W_2(p)}{1+W_1(p)W_2(p)K_1} \right] U(p)$$

$$\Rightarrow E_U(p) = \left[1 - \frac{K_2(p+1)}{p(p^2 + p + 1) + K_1K_2(p+1)} \right] U(p)$$

$$E_F(p) = 0 - Y_F(p) = -\frac{W_2(p)}{1+W_1(p)W_2(p)K_1} F(p)$$

$$\Rightarrow E_F(p) = -\left[\frac{K_2(p+1)}{p(p^2 + p + 1) + K_1K_2(p+1)} \right] F(p)$$

According to the beginning of the article: $U(p) = 1/p$ and $F(p) = A$.

Then:

$$e_{xl} = \lim_{p \rightarrow 0} pE_U(p) + \lim_{p \rightarrow 0} pE_F(p)$$

$$\lim_{p \rightarrow 0} pE_U(p) = \lim_{p \rightarrow 0} \left[p - \frac{pK_2(p+1)}{p(p^2 + p + 1) + K_1K_2(p+1)} \right] \frac{1}{p} = 1 - \frac{K_2}{K_1K_2} = \frac{K_1 - 1}{K_1}$$

$$\lim_{p \rightarrow 0} pE_F(p) = \lim_{p \rightarrow 0} \left[\frac{-pK_2(p+1)}{p(p^2 + p + 1) + K_1K_2(p+1)} \right] A = 0$$

$$\text{So: } e_{xl} = \frac{K_1 - 1}{K_1}$$

If $K_1 = 1$ is chosen, then the system's steady-state error $e_{xl} = 0$.

Exercise 2. Suppose the transfer function for a closed system has the form:

$$W_k(p) = \frac{16}{p^2 + (0,8 + 16K)p + 16}$$

a) Determine the value of K so that the damping coefficient $\xi = 0.5$.

b) Find the overshoot δ , time t_δ and transient time when the input signal is a unit step function of $1(t)$ calculated according to the 2% and 5% standards.

The answer:

a) Determine the value of K so that the damping coefficient $\xi = 0.5$

$$W_k(p) = \frac{16}{p^2 + p(0,8 + 16K) + 16} = \frac{\omega_n^2}{p^2 + 2\xi\omega_n p + \omega_n^2}$$

$$\Rightarrow \omega_n^2 = 16 \rightarrow \omega_n = 4.$$

On the other hand, knowing $\xi = 0.5$ so:

$$0.8 + 16K = 2 \xi \omega_n \Rightarrow K = \frac{2\xi\omega_n - 0,8}{2} = 0,2.$$

b) Find the overshoot δ , time t_δ and transient time when the input signal is a function of $1(t)$ calculated according to the 2% and 5% standards

- Overshoot adjustment:

$$\delta\% = e^{\left(\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right)}.100 = e^{\left(\frac{0,5 \times 3,14}{\sqrt{1-0,5^2}}\right)} = 16,32\%$$

- Adjustable time t_δ :

$$t_\delta = \frac{\pi}{\omega_n \sqrt{1-\xi^2}} = \frac{3,14}{4\sqrt{1-0,5^2}} = 1,04 \text{ (s)}$$

- Transit time:

+ According to 2% standard: $t_s = \frac{4}{\xi\omega_n} = \frac{4}{0,5 \times 4} = 2 \text{ (s)}$

+ According to 5% standard: $t_s = \frac{3}{\xi\omega_n} = \frac{3}{0,5 \times 4} = 1,5 \text{ (s)}$.

4.3.2. Self-explanatory exercises

1. Determine the position, velocity and acceleration coefficients of the step function, slope function, and parabolic function of the unit negative feedback control system with the following open system transfer function:

a) $W(p) = \frac{K}{p(1+0,1p)(1+0,5p)}$

b) $W(p) = \frac{1000}{p(10+p)(100+p)}$

Answer:

a) $K_p = \infty$; $K_v = K$; $K_a = 0$.

b) $K_p = \infty$; $K_v = 1$; $K_a = 0$.

2. For the control system described in Figure 4.23.

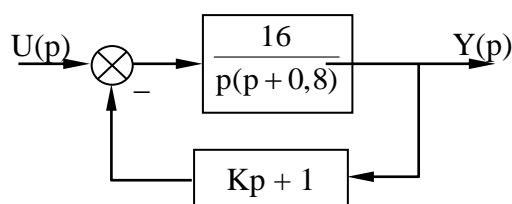


Figure 4.23. Automatic control system

- a) Determine the value of K so that the damping coefficient $\xi = 0.5$.
- b) Find the overshoot and transient time when the input signal is a unit step function $1(t)$ calculated according to two standards 2% and 5%.

Answer:

a) $K = 0.2$.

b) $\delta = 16.318\%$; $T_{qd} = 2s$ (standard 2%); $T_{qd} = 1.5s$ (standard 5%)

3. Given a system with unit negative feedback, know the open system transfer function is $W(p) = \frac{2K + 1}{p(p^2 + 2Tp + 1)}$. In which $K, T > 0$ and $\tau = 0.5$.

Please determine the quality integrals.

4. A ground radar used to detect airborne targets. Structure diagram in the azimuthal plane of the radar is shown in Figure 4.24.

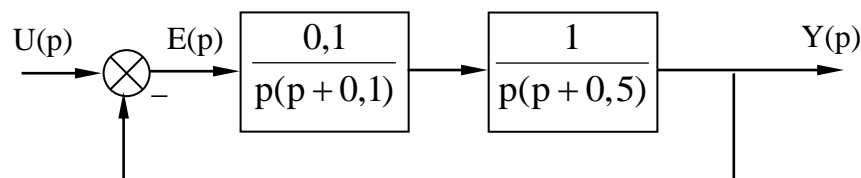


Figure 4.24. Structure diagram in the azimuthal plane of the radar

Determine the steady-state error of the system when the input signal is:

a) $u(t) = 10t$.

b) $u(t) = 10t + 6t^2$.

5. For the turret control system on the tank, have the diagram shown in Figure 4.25.

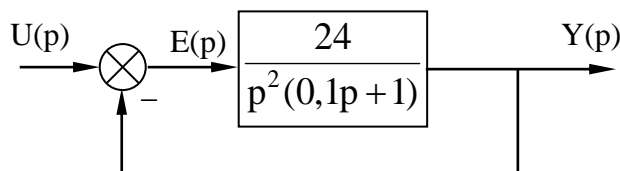


Figure 4.25. Turret control system on tanks

Determine the steady-state error of the system when the input signal is:

a) $u(t) = 100t$.

b) $u(t) = 12t^2$.

c) $u(t) = 20 + 100t + 12t^2$.

Answer: $W_E(p) = \frac{0.1p^3 + p^2}{0.1p^3 + p^2 + 24}$

a) $e(t) = 0$.

b) $e(t) = 0.041.24 = 0.984 = e_a$

c) $e(t) = 0 + 0 + 0.041.24 + 0 = 0.984 = e_a$.

6. The US military's Boeing V-22 is both a jet and a helicopter. The engine and propeller have the ability to rotate 90 degrees vertically and can change flight mode from helicopterto jet within 16 seconds. The simple structure diagram of the aircraft is shown in Figure 4.26.

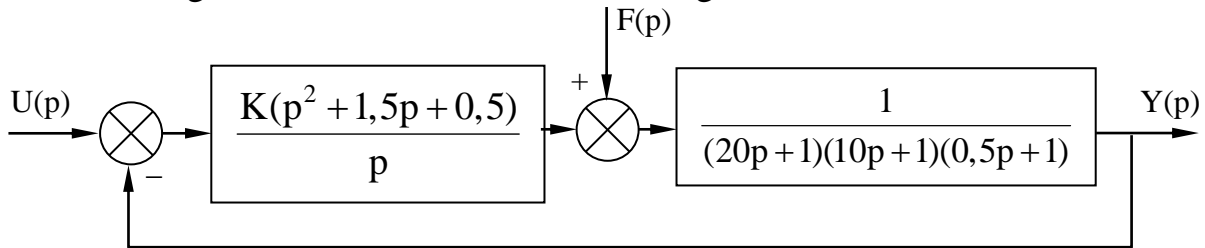


Figure 4.26. Simple structural diagram of the Boeing V-22 aircraft

- Determine K for the system to be stable.
- Determine δ and T_{qd} according to $h(t)$ when $K=280$ and there is no noise.
- Determine δ and T_{qd} in terms of $h(t)$ when $K = 280$ and there is a filter at the input equal to:

$$W_L(p) = \frac{0,5}{p^2 + 1,5p + 0,5}$$

Answer:

$$a) W(p) = K \frac{p^2 + 1,5p + 0,5}{100p^4 + 215p^3 + 30,5p^2 + p}$$

For the system to be stable: $K > 127,655$.

$$b) W_k(p) = \frac{280(p^2 + 1,5p + 0,5)}{100p^4 + 215p^3 + 310,5p^2 + 421p + 140}$$

$\delta > 0.85$ and $T_{qd} \approx 50s$.

$$c) W_k(p) = \frac{140}{100p^4 + 215p^3 + 310,5p^2 + 421p + 140}$$

$\delta > 0.08$ and $T_{qd} \approx 25s$.

7. Let the control system have the diagram as shown in Figure 4.27.

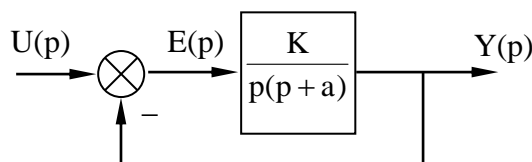


Figure 4.27. Automatic control system

- Find K , a for damping coefficient $\xi = \frac{1}{\sqrt{2}}$ and $\omega_n = 5\text{rad/s}$.
- Calculate $y(t)$ with K and a calculated in the previous section; $u(t) = 1(t)$.

8. The space telescope has a structural diagram as shown in Figure 4.28, in which K and K_1 are the coefficients that need to be chosen to achieve static and dynamic quality.

a) Simplify the structure diagram and write the transfer function for input signal $x(t)$ and noise $f(t)$:

$$Y(p)/X(p) \text{ and } Y(p)/F(p).$$

b) Determine the steady-state error $E(p)$ when the input signal is:

$$x(t) = 1(t) \text{ and } x(t) = t.$$

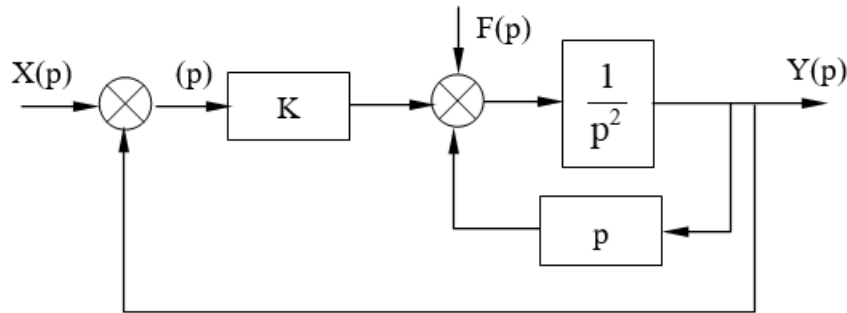


Figure 4.28. Space telescope control system

Answer:

$$a) \frac{Y(p)}{X(p)} = \frac{K}{p(p + K_1) + K}; \quad \frac{Y(p)}{F(p)} = \frac{1}{p(p + K_1) + K}$$

$$b) E(p) = \frac{p(p + K_1)}{p(p + K_1) + K} \cdot X(p)$$

9. Astronaut Bruce Mc Candless used an impulse device with a compressed air nozzle to move in space. The control mechanism is implemented by K_2 amplification as depicted in figure 4.29. The inertia of people and equipment is equal to $J = 25\text{kg}\cdot\text{m}^2$, K_3 has the function of reducing static deviation e_{st} . The coefficients K_1 , K_2 , K_3 help determine the static and dynamic quality of the system.

a) Write the transfer function of the closed system.

b) Determine $E(p)$ when the input is $x(t) = \varphi_0(t) = 1$.

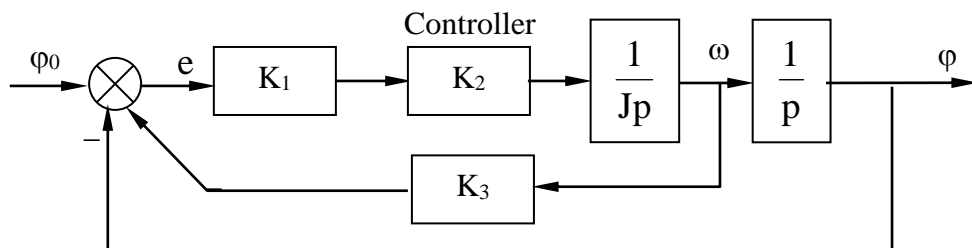


Figure 4.29. The control system sends people into space

Answer:

$$a) W_k(p) = \frac{\varphi(p)}{\varphi_0(p)} = \frac{K_1 K_2}{p(Jp + K_1 K_2 K_3) + K_1 K_2}$$

$$b) W_E(p) = \frac{E(p)}{\varphi_0(p)}; \varphi_0 = \frac{1}{p}; E(p) = \frac{Jp + K_1 K_2 K_3}{p(Jp + K_1 K_2 K_3) + K_1 K_2}$$

10. Figure 4.30 is a block diagram of the aircraft's altitude control system. Assuming the time constant T of the controller is 3s and the torque to inertia ratio K/J is $2/9 \text{ rad}^2/\text{s}^2$, find the damping coefficient of the system.

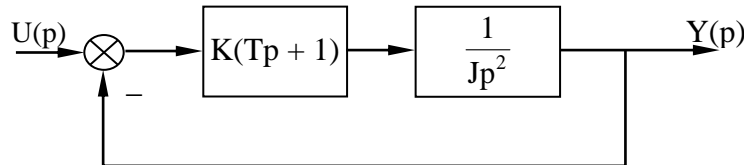


Figure 4.30. Aircraft altitude control system

11. The block diagram of the position control system of a large microwave antenna is depicted in Figure 4.31. To design such a system, we must account for the disturbance caused by the rotational force of wind gusts. Determine the value range of K_1 and K_2 so that the influence of noise $f(t) = \delta(t)$ is minimal in the steady state, for an input signal of $r(t) = 1(t)$.

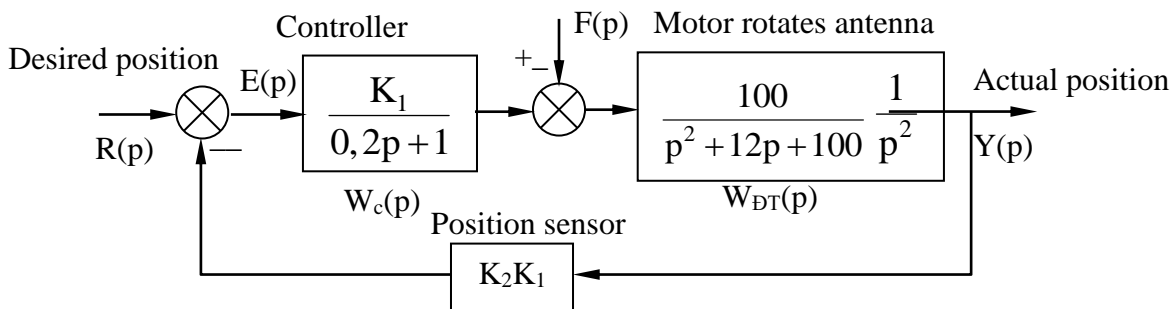


Figure 4.31. Ultra short wave antenna position control system

12. Block diagram of servo mechanism used in aircraft hydraulic systems, depicted in figure 4.32. Determine the steady state error when the input is $r(t) = a_0 + a_1t + a_2t^2$

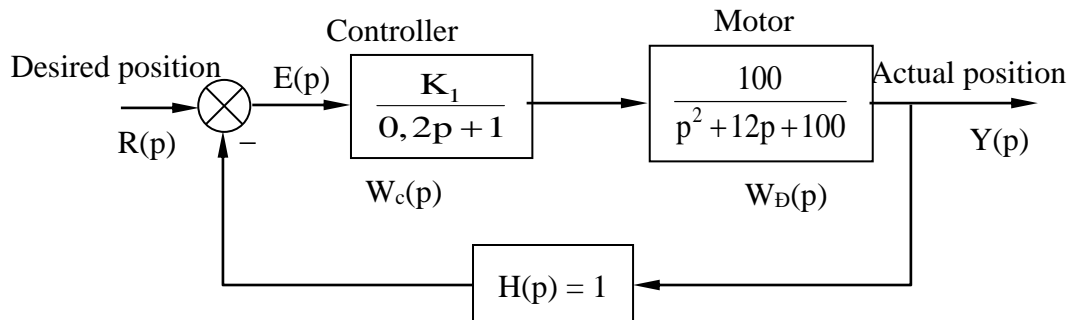


Figure 4.32. Servo mechanism in aircraft hydraulic systems

Conclusion of chapter 4

Even when a system achieves stability, it is imperative to assess its operational quality. This evaluation relies on specific criteria that serve as the basis for a thorough examination. These criteria constitute specific quality parameters. Leveraging the process parameters, encompassing both the transitional and establishment processes, individuals can gauge the system's quality level and devise strategies to either maintain or enhance the quality of the system.

Chapter 4 review questions

Question 1: Provide a comprehensive overview of the quality of the automatic control system throughout the transition and steady-state processes.

Question 2: Explain the method for determining errors in steady mode. How can one calculate the steady-state error with different inputs?

Question 3: Define the quality of the system during the transition process. Elaborate on the method for evaluating the quality of automatic control systems through quality integration.

Chapter 5

ANALYZING CONTROL SYSTEMS USING MATLAB

5.1. ANALYZING CONTROL SYSTEMS ACCORDING TO TIME CHARACTERISTICS USING MATLAB

5.1.1. Survey the stability of the control system according to time characteristics using Matlab

a) Survey steps

Step 1: Set the request transit time, SettlingTime_Req

Step 2: Determine information from response time

+ Determine the transfer function of the control system: Use the function tf(num, den) when knowing the polynomials of the transfer function or ss2tf(A, B, C, D) when knowing the state equation of the system control.

+ Create time responses such as transient responses and impulse transient responses. Use the functions step (num, den) and impulse (num, den).

+ Get detailed information about the control system's transient time.

```
S = stepinfo()
```

```
SettlingTime = S. SettlingTime
```

Step 3: Evaluate the stability of the control system

```
if SettlingTime <=inf
```

```
disp('System is stable');
```

```
elseif SettlingTime >=inf
```

```
disp('System is unstable');
```

```
end
```

```
if SettlingTime <= SettlingTime_Req
```

```
disp('System stabilized quickly');
```

```
else. else
```

```
disp('System stabilizes slowly');
```

```
end
```

b) Example 5.1

Let the control system have the following transfer function:

$$\text{sys} = \frac{5}{s^2 + 2s + 3}$$

Investigate the stability of the control system according to time characteristics.

+ The calculation and survey program are built as follows:

```

% Step 1: Set the required transit time
SettlingTime_Req = 5      % Request transient time
% Step 2: Determine information from response time
% Determine the equation of the state of the control system
num = [5];
den = [1, 2, 3];
sys = tf(num, den)
% Generates timing responses and retrieves response information
step(sys);
S = stepinfo(sys);
SettlingTime = S.SettlingTime
% Step 3: Evaluate the stability of the control system
% Check system stability
if SettlingTime <=inf
disp('System is stable');
elseif SettlingTime >=inf
disp('System is unstable');
end
if SettlingTime <= SettlingTime_Req
disp('System stabilized quickly');
else. else
disp('System stabilizes slowly');
end

```

+ After running the program, the results received are the transient time, assessment of system stability and transient characteristics of the system (Figure 5.1).

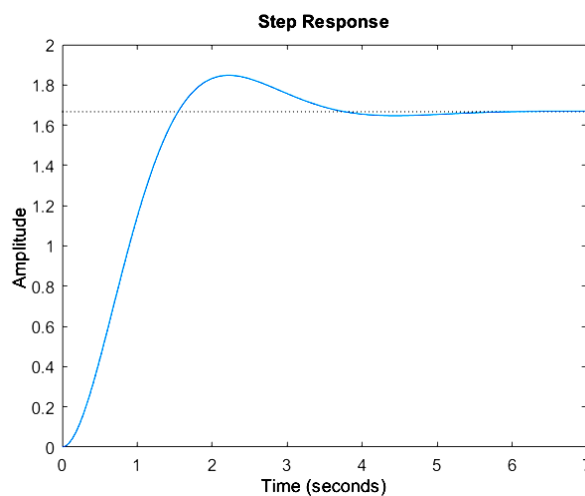


Figure 5.1. System transient characteristics

SettlingTime_Req = 5

5

sys = -----

$s^2 + 2s + 3$

SettlingTime = 3.4043

The system stabilizes quickly

5.1.2. Survey the quality of the system according to time characteristics using Matlab

a) Survey steps

Step 1. Set required quality indicators and declare communication functions

Call the function “HTDK_YeuCau” to load input data. The code for this function is as follows:

```
function HTDK_YeuCau()
disp('Enter required quality indicators:');
% Quality indicator of the system during the transition process
maxOvershoot = input('Overshoot threshold (%), Example 10: ');
maxRiseTime = input('Rise time threshold (s), example 1: ');
maxSettlingTime = input('Transient time threshold (s), example 5: ');
% Quality indicator of the system in steady mode
maxErrorSteadyState = input('Static error threshold, eg 0.1: ');
maxErrorVelocity = input('Error threshold based on speed, eg 0.2: ');
maxErrorAcceleration=input('Acceleration error threshold, example 0.3: ');
% Determine the transfer function of the control system.
disp('Enter the polynomial of the transfer function :');
num = input('Polynomial numerator, eg [1]: ');
den = input('Polynomial denominator, eg [1 2 1]: ');
sys = tf(num, den); % Create transfer function
% Populate values into workspace
assignin('base', 'maxOvershoot', maxOvershoot);
assignin('base', 'maxRiseTime', maxRiseTime);
assignin('base', 'maxSettlingTime', maxSettlingTime);
assignin('base', 'maxErrorSteadyState', maxErrorSteadyState);
assignin('base', 'maxErrorVelocity', maxErrorVelocity);
assignin('base', 'maxErrorAcceleration', maxErrorAcceleration);
```



```

assignin('base', 'num', num);
assignin('base', 'den', den);
assignin('base', 'sys', sys);
end

```

Step 2 . Determine information from response time

- Create time responses such as transient response and impulse transient response: Use the function `step(num, den)`, `impulse(num, den)`.

```

% Plot the step response
step(sys)
xlabel('Time (s)');
ylabel('System output');
title('Step response of the control system');

```

- Get detailed information about response time during transients
system quality indicators during the transition process

```

info = stepinfo(sys);
disp('System quality information during transition:');
disp(['RiseTime: ' num2str(info.RiseTime) ' seconds']);
disp(['SettlingTime: ' num2str(info.SettlingTime) ' seconds']);
disp(['Maximum overshoot: ' num2str(info.Overshoot) ' %']);
% Calculate the set value
steadyStateValue = y(end);
disp(['Stable value: ' num2str(steadyStateValue)]);

```

- Get detailed information about the time response in steady state mode, including steady error types such as Static error , Speed error , Acceleration error using the function to determine the steady error constants Establishment and static order, Function to determine the stability errors.

+ Function to determine the steady-state error constants and static orders of the system:

```

function Hang_So_Sai_So(num, den)
% Example Hang_So_Sai_So([0.5], [1 0.2 10])
% Check if the transfer function is valid
if isempty(num) || isempty(den)
disp('Invalid transfer function .');
return;
end
rt = tf(num, den);

```

```

syms sy; % Symbolic variable (use another name)
num = poly2sym(num, sy);
den = poly2sym(den, sy);
gs = num / den; % Symbolic representation of the transfer function
Kp = lim(gs); % Position constant, Kp
Kv = lim(sy * gs); % Velocity constant, Kv
Ka = lim(sy^2 * gs); % Acceleration constant, Ka
fprintf('\n Establishment error constants:\n\t\t Kp = %.3f,\n\t\t Kv = %.3f,\n\t\t Ka
= %.3f\n ', Kp, Kv, Ka);
checkSystemType(Kp, Kv, Ka);
disp(' '); % Down the line
end
%% User-defined function
function y = lim(f)
syms sy;
y = limit(f, sy, 0);
if isnan(y)
y = inf;
end
end
function checkSystemType(Kp, Kv, Ka)
if ~isinf(Kp) && Kv == 0 && Ka == 0
disp('System static level: 0');
elseif isinf(Kp) && ~isinf(Kv) && Ka == 0
disp('System static degree: 1');
elseif isinf(Kp) && isinf(Kv) && ~isinf(Ka)
disp('System static degree: 2');
end
end
+ Function to determine established errors
function Sai_So(num, den)
% Example False_So([0.5], [1 0.2 10])
% Check if the transfer function is valid
if isempty(num) || isempty(den)
disp('Invalid transfer function. ');
return;

```

```

end
sys = tf(num, den);
r=1;    % Input quantity
[y,t] = step(sys); % System response to step set value
step(sys);
% Static error
e_steady_state = r - y(end)
% Error by speed
s = tf('s');
Ys = sys*(1/s); %Laplace image output quantity
Rs = tf([r], [1]); % Laplace image set quantity
e_velocity = evalfr(s*(1-Ys/Rs),0)
% Error according to acceleration
e_acceleration = evalfr(s^2*(1-Ys/Rs),0)
% Show results
disp(['Static error: ' num2str(e_steady_state)]);
disp(['Velocity error: ' num2str(e_velocity)]);
disp(['Acceleration error: ' num2str(e_acceleration)]);

```

Step 3 . Evaluate the quality of the control system

The program evaluates the quality of the system, as follows:

```

function Chat_Luong (num, den)
% Example Chat_Luong([0.5], [1 0.2 10])
% Check if the transfer function is valid
if isempty(num) || isempty(den)
disp('Invalid transfer function. ');
return;
end
sys = tf(num, den);
[y, t] = step(sys); % Gets the system response
% Plot the step response
step(sys)
xlabel('Time (s)');
ylabel('System output');
title('Step response of the control system');
system quality indicators during the transition process
info = stepinfo(sys);

```

```

disp('System quality information during transition:');
disp(['Increase time(Rise time):' num2str(info.RiseTime) 'seconds']);
disp(['Settling time:' num2str(info.SettlingTime)'second']);
disp(['Maximum overshoot(Overshoot):' num2str(info.Overshoot) ' %']);
% Calculate steady-state value
steadyStateValue = y(end);
disp(['Steady-state value: ' num2str(steadyStateValue)]);
% Evaluate the quality of the system during the transition process
if info.RiseTime <= maxRiseTime
disp('System responds quickly');
else. else
disp('System response is slow');
end
if info.Overshoot > maxOvershoot
disp('The system has high overcorrection');
else. else
disp('The system has low overshoot');
end
% Calculate the established errors
% static error
r=1; %Input quantity
e_steady_state = r - y(end);
% Error by speed (velocity error)
s = tf('s');
Ys = sys*(1/s); %Laplace image output quantity
Rs = tf([r], [1]); % Laplace image set quantity
e_velocity = evalfr(s*(1-Ys/Rs),0);
% Acceleration error
e_acceleration = evalfr(s^2*(1-Ys/Rs),0);
% Show results
disp(['Static error: ' num2str(e_steady_state)]);
disp(['Velocity error: ' num2str(e_velocity)]);
disp(['Acceleration error: ' num2str(e_acceleration)]);
% Evaluation of established errors
disp('Evaluate quality in steady mode:');
if abs(e_steady_state) > maxErrorSteadyState

```

```

disp('The system has high static error');
else. else
disp('System has low static error');
end
if abs(e_velocity) > maxErrorVelocity
disp('The system has high speed error');
else. else
disp('The system has low speed error');
end
if abs(e_acceleration) > maxErrorAcceleration
disp('The system has high acceleration error');
else. else
disp('The system has low acceleration error');
end

```

b) Example 5.2

Let the transfer function of the antenna control system be as follows

$$W = \frac{s + 12}{s^2 + 15s + 54}$$

Calculate the steady-state errors , error constants and evaluate the quality of the control system.

- Call the function: “ *HTDK_YeuCau* ” to load input data. The following results:

```
>> HTDK_YeuCau
```

Enter the required quality indicators:

Overcorrection threshold (%), Example 10: 10

Threshold rise time (s), Example 1:1

Transient time threshold (s), Example 5: 5

Static error threshold, example 0.1: 0.1

Error threshold based on speed, example 0.2: 0.2

Error threshold according to acceleration, example 0.3: 0.3

Enter the polynomial of the transfer function :

Polynomial numerator, example [1]: [1 12]

Polynomial denominator, for example [1 2 1]: [1 15 54]

$$\text{sys} = \frac{s + 12}{s^2 + 15s + 54}$$

- Call function Sai_So(num, den). The following results:
 - Static error: 0.77877
 - Error according to speed: Inf
 - Acceleration error: Inf
- Call the function Hang_So_Sai_So (num, den). The following results:
 - Establishment error constants:
 - $K_p = 0.222,$
 - $K_v = 0.000$
 - $K_a = 0.000$
 - System static level: 0
- Call the function Chat_Luong(num, den). The following results:
 - + System quality information during transition:
 - Rise time: 0.41258 seconds
 - Settling time: 0.71292 seconds
 - Overshoot: 0 %
 - Steady-state value: 0.22123
 - + Evaluate the quality of the system during the transition process:
 - The system responds quickly
 - The system has low overcorrection
 - The system stabilizes quickly
 - + System quality information in steady mode:
 - Static error: 0.77877
 - Error according to speed: Inf
 - Acceleration error: Inf
 - + Quality assessment in steady mode:
 - The system has high static errors
 - The system has high speed errors
 - The system has high acceleration errors

5.2. ANALYSIS OF CONTROL SYSTEMS IN THE FREQUENCY DOMAIN USING MATLAB

5.2.1. Survey the stability of the control system in the frequency domain using MatLab

a) Survey steps

Step 1. Determine the transfer function of the system in the frequency domain.

The system's transfer function in the frequency domain ($G(j\omega)$) represents the relationship between the input and output of the control system over frequency. This can be done by performing measurement experiments or system simulations in the frequency domain.

Step 2 . Draw the frequency characteristics of the system.

From the transfer function, draw the system's frequency characteristics, such as: Frequency phase amplitude characteristics, Bode, and Nyquist plots. Frequency characteristics represent the system's response to input signals at different frequencies.

Step 3 . Analyze frequency characteristics.

Analyze frequency characteristics to find important parameters such as edge cutoff frequency, phase cutoff frequency, amplitude and phase reserve,...

Step 4 . Determine system stability.

Based on the results of the frequency characteristic analysis, the stability of the system can be determined. If the system has positive amplitude margin and phase margin, then the system is considered stable and is able to tolerate larger variations of the input signal. Conversely, if the amplitude margin or phase margin has a negative value, the system may be near the instability limit and susceptible to uncontrolled oscillations.

In Matlab, to evaluate the stability of the control system in the frequency domain, the following functions can be used:

Function `sys = tf(num, den)`: Used to create the transfer function of the control system based on system coefficients, such as proportional coefficients, time constants, natural frequencies,...

Function `bode(sys)`: used to draw the Bode chart of the system.

Function `nyquist(sys)`: used to draw the Nyquist chart of the system.

`Margin(sys)` function: used to calculate the amplitude and phase margin of the system, and the margin and phase cutoff frequencies.

b) Example 5.3

Give the structural diagram of a flight control system (Figure 5.2). Find the criteria to evaluate the stability in the frequency domain of the above system.

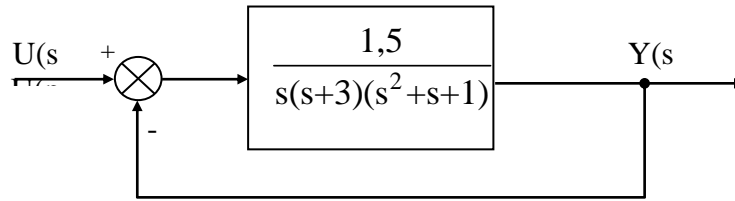


Figure 5.2. Aircraft flight control system

- In Matlab, you can use the functions `tf`, `bode`, `nyquist`, `margin`, to write an mfile to calculate indicators to evaluate the stability in the frequency domain of the control system .

```
clc, clear all,
```

```
% Create the system's frequency transfer function
```

```
num = [1.5]; % Numerator
```

```
den = conv(conv([1 0], [1 3]), [1 1 1]); % Denominator
```

```
sys = tf(num, den);
```

```
% Draw Bode, nyquist plots
```

```
nyquist(sys);
```

```
figure
```

```
margin(sys)
```

% Calculate amplitude and phase margin, cutoff frequency and phase cutoff frequency

```
[Gm,Pm,Wcg,Wcp] = margin(sys);
```

```
disp(['Amplitude reserve: ', num2str(20*log10(Gm)), ' dB']);
```

```
disp(['Phase reserve degrees: ', num2str(Pm), ' degrees']);
```

```
disp(['Cutting frequency: ', num2str(Wcg), ' rad/s']);
```

```
disp(['Phase cutoff frequency: ', num2str(Wcp), ' rad/s']);
```

+ The result after running the program will be the Bode and nyquist charts of the system (Figure 3.8) and indicators to evaluate the stability in the frequency domain of the system as follows:

Amplitude reserve: 4.2169 dB

Phase reserve: 40.89 degrees

Cutoff frequency: 0.86602 rad/s

Phase cutoff frequency: 0.5541 rad/s

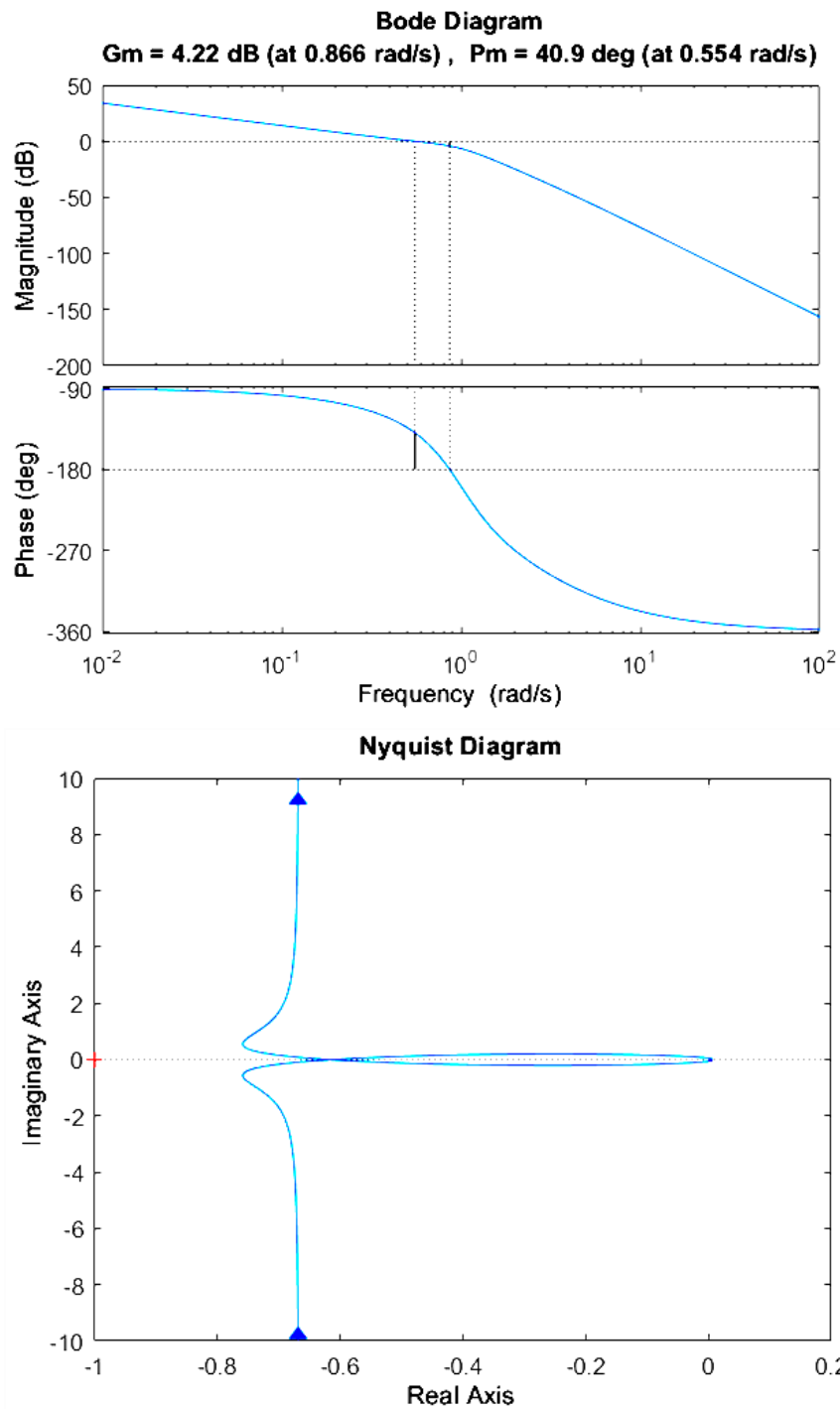


Figure 5.3. Bode, nyquist plots of the flight control system

3.2.2. Survey the quality of the control system in the frequency domain using Matlab

a) Survey steps

Step 1. Determine the transfer function of the system in the frequency domain $G(j\omega)$.

Step 2. Draw the frequency characteristics of the system such as Frequency Phase Amplitude Characteristics, Bode and Nyquist Plots.

Step 3. Analyze frequency characteristics.

Analyze frequency characteristics to find important parameters such as cutoff frequency, phase cutoff frequency, amplitude reserve, phase reserve , resonance peak, resonance frequency, slope, bandwidth of the system.

Step 4. Determine required quality goals.

Determining required quality objectives is the process of determining the standards that the control system needs to meet to achieve the desired quality:

+ Required slope: The goal is to ensure the system is able to respond quickly to frequency changes.

+ Required amplitude and phase margin: The goal is to ensure the system has high stability under variable conditions.

+ Required resonance peak: The goal is to have a suitable resonance peak value to maintain stable response and quality of the system.

+ Required bandwidth: The goal is to have the appropriate bandwidth to ensure the system has the best signal transmission performance.

Step 5 . Evaluate the quality of the control system.

Compare quality indicators with the required quality goals. If the quality indicators meet the required quality objectives, the system can be considered to have achieved the desired quality. If there is a discrepancy between quality indicators and the required goals, it is necessary to make adjustments and improve the system to achieve the desired quality.

- In Matlab, to evaluate the quality of the control system in the frequency domain, the following functions can be used:

Function `sys = tf(num, den)`: Used to create the transfer function of the control system based on system coefficients, such as proportional coefficients, time constants, natural frequencies,...

Function `bode(sys)`: used to draw the Bode chart of the system.

Function `nyquist(sys)`: used to draw the Nyquist chart of the system.

`Margin(sys)` function: used to calculate the amplitude and phase margin of the system, and the margin and phase cutoff frequencies.

b) Example 5.4

Given a control system with the following transfer function :

$$\text{sys} = \frac{1}{s^2 + 2s + 4}$$

Let's evaluate the quality of the control system in the frequency domain compared to the required quality target: Required amplitude margin 20 dB and required phase margin 40 deg.

In Matlab, you can use the functions `tf`, `bode`, `nyquist`, `margin`, to write an mfile to calculate indicators to evaluate the quality of the control system in the frequency domain .

```

clc, clear all

% Step 1: Determine the transfer function of the system in the frequency
domain G(j $\omega$ )
num = [1];
den = [1, 2, 4];
sys = tf(num, den)
% Step 2: Draw the system's frequency characteristics
nyquist(sys) % Nyquist plot
title('Nyquist chart')
grid on
figure
margin(sys) % Bode plot
% Step 3: Analyze frequency characteristics
% amplitude reserve, phase reserve
[Gm, Pm, Wcg, Wcp] = margin(sys);
% Convert amplitude margin to dB
peakGain = 20 * log10(Gm);
% Calculate the edge cutoff frequency and phase cutoff frequency
[~, ~, wc] = bode(sys);
crossoverFrequency = wc(1); % Cutoff frequency (rad/s)
crossoverPhase = wc(2); % Phase cutoff frequency (rad/s)
% Calculate Resonance Peak and Resonance Frequency
wn = sqrt(den(end)); % Natural Frequency
zeta = sqrt(1 / (4 * den(1))); % Damping coefficient
peakFrequency = wn * sqrt(1 - 2 * zeta^2); % Resonance frequency (rad/s)
peakAmplitude = 1 / (2 * zeta * sqrt(1 - zeta^2)); % Resonance peak
% Calculate the slope
w_high = crossoverFrequency;
w_low = crossoverFrequency / 10; % Lower frequency
[mag_high, ~] = bode(sys, w_high); % Value at cutoff frequency
[mag_low, ~] = bode(sys, w_low); % Value at lower frequency
slope = (20 * log10(mag_high) - 20 * log10(mag_low)) / (w_high - w_low); %
Slope (dB/decade)
% Calculate bandwidth

```

```

bandwidth = bandwidth(sys);
% Print parameters
disp(['Crossover frequency: ', num2str(crossoverFrequency), ' rad/s']);
disp(['Phase cutoff frequency: ', num2str(crossoverPhase), ' rad']);
disp(['Amplitude margin: ', num2str(peakGain), ' dB']);
disp(['Phase margin: ', num2str(Pm), ' deg']);
disp(['Resonance peak: ', num2str(peakAmplitude)]);
disp(['Resonance frequency: ', num2str(peakFrequency), ' rad/s']);
disp(['System slope: ', num2str(slope), ' dB/decade']);
disp(['Bandwidth: ', num2str(bandwidth), ' rad/s']);
% Step 4: Determine required quality goals
desiredPeakGain = 20; % Required amplitude margin (dB)
desiredphase = 20; % Required phase margin (deg)
disp(['Required phase margin: ', num2str(desiredphase), 'deg']);
disp(['Requested PeakGain: ', num2str(desiredPeakGain), 'dB']);
% Step 5: Evaluate the quality of the control system (can be added)
if Pm >= desiredphase && peakGain >= desiredPeakGain
disp('The system meets the desired quality.')
else. else
disp('The system needs adjustment to achieve the desired quality.')
end

```

+ The result after running the program will be the Bode and nyquist charts of the system (Figure 3.9), (Figure 3.10) and the required quality indicators and quality assessment of the control system as follows:

Cutoff frequency: 0.1 rad/s

Phase cutoff frequency: 0.10229 rad

Amplitude margin: Inf dB

Phase reserve: Inf deg

Resonance peak: 1.1547

Resonance frequency: 1.4142 rad/s

System slope: 0.11928 dB/decade

Bandwidth: 2.5424 rad/s

Required phase margin: 40 deg

Required amplitude margin: 20 dB

The system meets the desired quality.

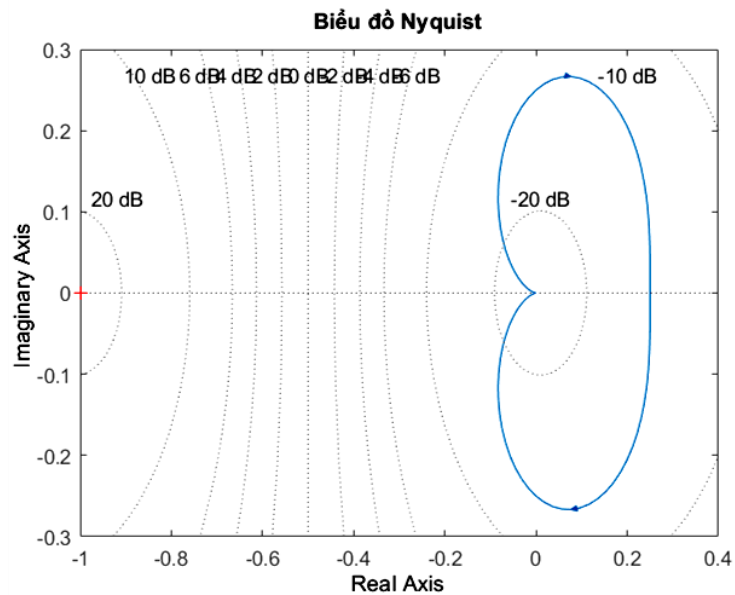


Figure 5.4. Nyquist plot of the system

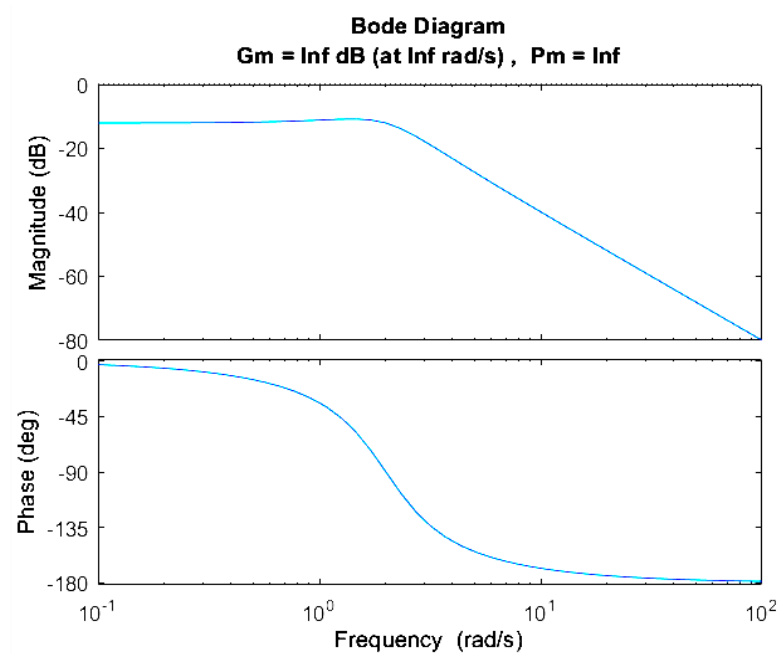


Figure 5.5. Bode plot of the system

5.3. CHAPTER 5 EXERCISES

5.3.1. Sample solution exercises

Exercise 1. Write a Function file function to automatically evaluate the stability and quality of a control system in the frequency domain.

Applied to a control system with the following transfer function

$$\text{sys} = \frac{4}{s^2 + 4s + 1}$$

Know the required quality target: Required amplitude margin 20 dB and required phase margin 60 deg.

- The evaluateControlSystem function below will automatically evaluate the stability and quality of the control system in the frequency domain, when this function is called with the system parameters and required quality goals.

```
function evaluateControlSystem(num, den, Gyc, Pyc)
% Step 1: Determine the transfer function of the system in the frequency
domain G(j $\omega$ )
sys = tf(num, den);
% Step 2: Draw the system's frequency characteristics
figure
nyquist(sys) % Nyquist plot
title('Nyquist chart')
grid on
figure
margin(sys) % Bode plot
% Step 3: Analyze frequency characteristics
% amplitude reserve, phase reserve
[Gm, Pm, ~, ~] = margin(sys);
% Convert amplitude margin to dB
peakGain = 20 * log10(Gm);
% Print parameters
disp(['Amplitude margin: ', num2str(peakGain), ' dB']);
disp(['Phase margin: ', num2str(Pm), ' deg']);
% Step 4: Determine required quality goals
disp(['Required phase margin: ', num2str(Pyc), ' deg']);
disp(['Required amplitude margin: ', num2str(Gyc), ' dB']);
% Step 5: Evaluate the stability of the control system
if Gm > 0 && Pm > 0
disp('System is stable. ');
else. else
disp('The system is unstable or on the border of stability. ');
end
% Step 6: Evaluate the quality of the control system
if Pm >= Pyc && peakGain >= Gyc
disp('The system meets the desired quality. ')
else. else
disp('The system needs adjustment to achieve the desired quality. ')
end
end
```

Applied to the given control system, the evaluateControlSystem function needs to be called with the system parameters and required quality goals, as follows:

```
num = [1];
den = [1, 2, 4];
Gyc = 20; % Required amplitude margin (dB)
Pyc = 40; % Required phase margin (deg)
evaluateControlSystem(num, den, Gyc, Pyc);
```

- The following results:
 Amplitude margin: Inf dB
 Phase reserve: Inf deg
 Required amplitude margin: 20 dB
 Required phase margin: 40 deg
 System is stable.
 The system meets the desired quality.

Exercise 2. Use MatLab / Simulink to calculate the transfer function from input to output of the system with the structural diagram in Figure 5.6.

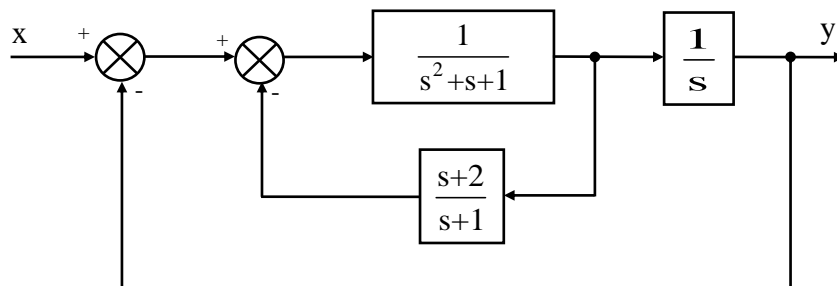


Figure 5.6 . Structure diagram of the control system

Instruct:

- Build a Simulink diagram of the control system as shown in Figure 5.7 with the name BaiTapGiaiMau.slx

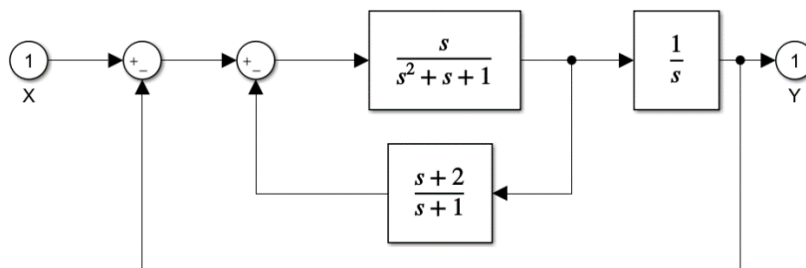


Figure 5.7 . Simulink diagram of the control system

- Run the following code:

```
[a,b,c,d] = linmod('BaiTapGiaiMau');
```

[num,den]= ss2tf(a,b,c,d);

H=tf(num,den)

The following results:

$$H = \frac{s^2 + s}{s^4 + 3s^3 + 5s^2 + 2s - 1.269e-16}$$

5.3.2. Self-explanatory exercises

1. Build a program or use a set of different MatLab functions to calculate the transfer function from input to output of systems with the structural diagram in Figure 5.8.

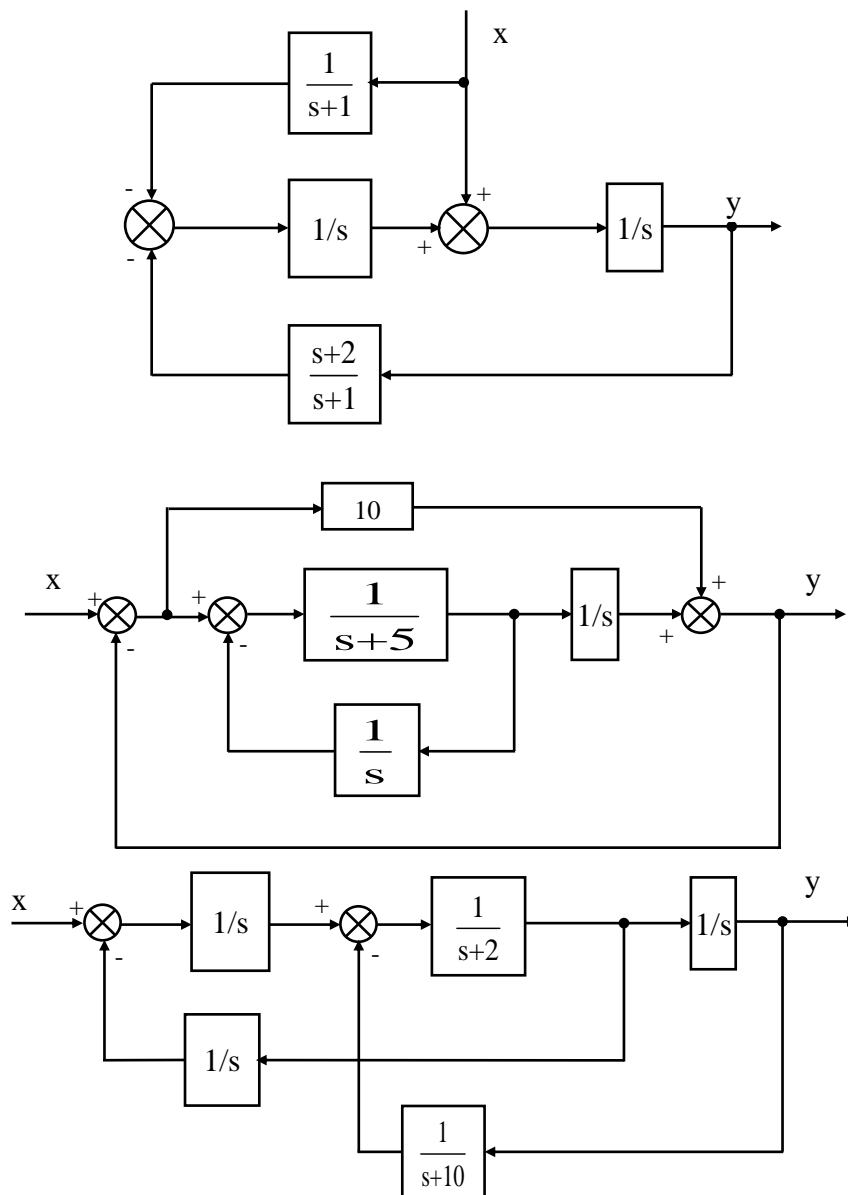


Figure 5 . 8 . Structure diagram of closed control loops

Instructions: Use the set of functions available in Matlab such as tf function, feedback function, and step function.

2. Use functions in MatLab to calculate the open-circuit transfer function, error-based transfer function, closed-circuit transfer function and perturbation transfer function $F(s)$ of linear control systems with primitives. The structure diagram is as shown in Figure 5.9.

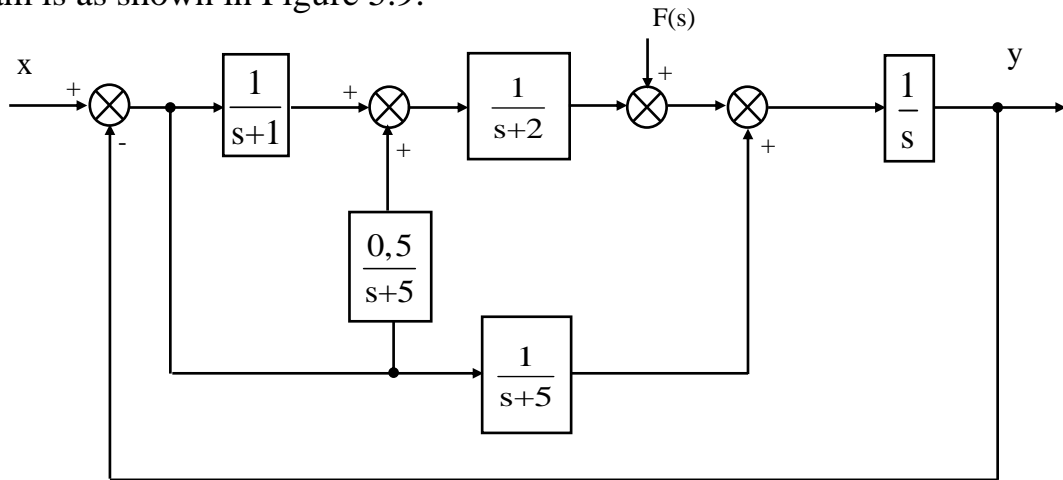


Figure 5.9. Structure diagram of linear control systems

Instruct :

To calculate the open-circuit transfer function , use the function `tf()`;

To calculate the closed-circuit transfer function, use the `feedback()` function;

To calculate the transfer function in terms of bias, use the functions `tf()`, `feedback()`;

To calculate the perturbation transfer function $F(s)$, use the functions `tf()`, `feedback()`.

3. The aircraft's direction control system has a structural diagram (Figure 5.10).

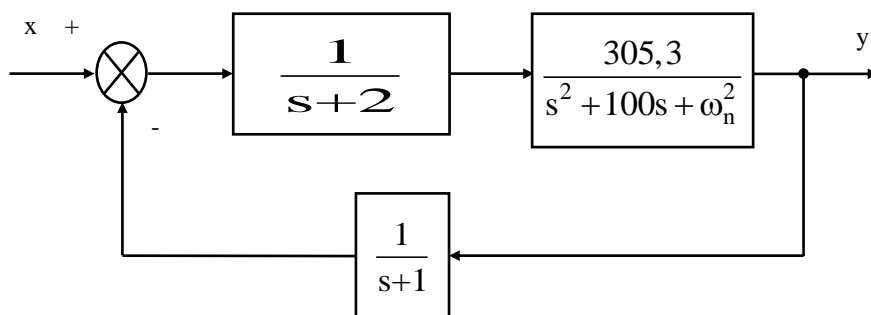


Figure 5.10. Structure diagram of the aircraft's direction control system

a) Construct a Bode plot of the above system and determine the phase and amplitude margin when $\omega_n^2 = 15.267$.

b) Repeat step a, when $\omega_n^2 = 9500$.

Instructions: Use the `tf` function to create the system's transfer function and `bode` to calculate the frequency response, use the `margin` function to calculate the system's Gain Margin (GM_dB) and Phase Margin (PM_deg).

4. Use Bode or Nyquist plots to find the K value that ensures the system is stable when knowing that the open-circuit transfer function of a unit negative feedback closed system has the following form:

$$a) G(s).H(s) = \frac{K(s+1)}{s(s+2)(s+5)(s+10)}$$

$$b) G(s).H(s) = \frac{K}{s(s+2)(s+0,1)(s+10)}$$

Instruct:

Construct an open-circuit transfer function, draw a Bode plot of the open-circuit transfer function, find the K value to ensure stability as follows:

[~, GM, ~, Wc] = margin(sys);

PM = 180 + angle(evalfr(sys, Wc)*1i); % Calculate phase margin (PM) at cutoff frequency (Wc)

K_stable = 1 / abs(evalfr(sys, Wc)*1i); % K value is guaranteed to be stable.

Chapter 5 review questions

Question 1. What is a complex transfer function? The effect of complex transfer functions in determining the frequency characteristics of the control system.

Question 2. Present the method of calculating the frequency amplitude characteristics and frequency phase characteristics of the system from the complex transfer function ?

Question 3. State the meaning and role of the Nyquist chart in evaluating the stability of a control system.

Question 4. How does the Nyquist plot $G(j\omega)$ affect the stability of the system?

Question 5. State the formula for calculating the Bode plot for the frequency amplitude and frequency phase characteristics of the complex transfer function .

Question 6. Explain how to calculate the resonance peak of a second-order control system?

Question 7. Write an expression to calculate the resonant frequency for a second-order control system?

Question 8. What are edge cutoff frequency and phase cutoff frequency? Write an expression to calculate these 2 parameters.

Question 9. What are amplitude margin and phase margin? Present how to evaluate the stability of the system based on amplitude reserve and phase reserve.

Question 10. Present the method to evaluate the quality of the control system in the frequency domain?

REFERENCES

1. Le Cong Luan, Automatic control theory, AD-AF of the Academy in Vietnam - 2023;
2. Nguyen Tang Cuong, Major exercises in the basic subject of automatic control theory, Military Technical Academy - 2013;
3. Le Anh Dung, Flight theory and air defense missile control system, Military Technical Academy - 2014;
4. Nguyen Thi Phuong Ha, Automatic control theory, Ho Chi Minh City National University Press - 2015;
5. Nguyen Thuong Ngo, Conventional and modern automatic control theory, Science and Technology Publishing House - 2013;
6. Vu Hoa Tien, Dynamics of aircraft control systems, Military Technical Academy - 2013;
7. Le Van Thiep, Automatic control theory, Air Defense Academy - 2010;
8. Katsuhiko Otaga, Modern Control Engineering, Prentice Hall, New Jersey, United States of America - 2009;
9. PN Paraskevopoulos, Modern Control Engineering, Marcel Dekker, New York, United States of America - 2002.

APPENDIX

Original-image table of some common functions

No.	Original function	Image function
1	$\delta(t)$	1
2	$1(t)$	$1/p$
3	At	$\frac{A}{p^2}$
4	$\frac{A}{(n-1)}t^{n-1}$	$\frac{A}{p^n}$
5	$A.e^{-\alpha t}$	$\frac{A}{p+\alpha}$
6	$\frac{A}{(n-1)!}t^{n-1}.e^{-\alpha t}$	$\frac{A}{(p+\alpha)^n}$
7	$A.\text{Sin}\omega t$	$\frac{A.\omega}{p^2+\omega^2}$
8	$A.\text{Cos}\omega t$	$\frac{A.p}{p^2+\omega^2}$
9	$\frac{A}{\omega}.e^{-\alpha\frac{t}{2}}\text{sin}\omega t; \omega = \sqrt{\beta^2 - \frac{\alpha^2}{4}}$	$\frac{A}{p^2+\alpha p+\beta^2}$
10	$A.e^{-\alpha\frac{t}{2}}(\text{Cos}\omega t - \frac{t}{2\omega}\text{sin}\omega t); \omega = \sqrt{\beta^2 - \frac{\alpha^2}{4}}$	$\frac{Ap}{p^2+\alpha p+\beta^2}$
11	$\frac{A}{\alpha}(1-e^{-\alpha t})$	$\frac{A}{p(p+\alpha)}$
12	$\frac{A}{\omega^2}(1-\text{Cos}\omega t)$	$\frac{A}{p(p^2+\omega^2)}$
13	$f'(t)$	$pF(p) - f(0)$
14	$f''(t)$	$p^2.F(p) - \frac{f(0)}{p} - \frac{f'(0)}{p^2}$
15	$\int_0^t f(t)dt$	$\frac{F(p)}{p}$

No.	Original function	Image function
16	t^n	$\frac{n!}{p^{n+1}}$
17	$t.e^{-at}$	$\frac{1}{(p+a)^2}$
18	$t^n.e^{-at}$	$\frac{n!}{(p+a)^{n+1}}$
19	$\frac{1}{a}(1 - e^{-at})$	$\frac{1}{p(p+a)}$
20	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{p^2(p+a)}$
21	$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(p+a)(p+b)}$
22	$\frac{1}{ab} \left[1 + \frac{1}{a-b}(be^{-at} - ae^{-bt}) \right]$	$\frac{1}{p(p+a)(p+b)}$
23	$e^{-at} \sin \omega t$	$\frac{\omega}{(p+a)^2 + \omega^2}$
24	$e^{-at} \cos \omega t$	$\frac{p+a}{(p+a)^2 + \omega^2}$
25	$\sin(\omega t + \varphi)$	$\frac{\omega \cos \varphi + p \sin \varphi}{p^2 + \omega^2}$
26	$\cos(\omega t + \varphi)$	$\frac{p \cos \varphi - \omega \sin \varphi}{p^2 + \omega^2}$
27	$e^{-at} \sin(\omega t + \varphi)$	$\frac{\omega \cos \varphi + (p+a) \sin \varphi}{(p+a)^2 + \omega^2}$
28	$\frac{1}{\omega^2} - \frac{e^{\xi \omega t}}{\omega^2 \sqrt{1-\xi^2}} \sin(\omega \sqrt{1-\xi^2} . t + \varphi)$	$\frac{1}{p(p^2 + 2\omega \xi p + \omega^2)}$